Instituto Superior de Economia e Gestão
UNIVERSIDADE TÉCNICA DE LISBOA
DESDE 1911

Master in Actuarial Science
Loss Reserving
24-06-2014
Time allowed: 2 hours

Instructions:

1. This paper contains **6** questions and comprises **8** pages including the title page and **3** preprinted answer sheets.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.
11. Preprinted answer sheets are available for some of the tables required.

You are the actuary valuing the claim cost of a portfolio of liability insurance. This is the information you have:

Accident year	Earned premium
2007	2 339
2008	2 773
2009	3 039
2010	2 947
2011	3 440
2012	3 707
2013	4 615

Cumulative reported claim numbers

	Development year						
Accident year	0	1	2	3	4	5	6
2 007	57	102	111	112	113	113	114
2 008	76	131	137	142	144	145	
2 009	114	156	165	168	168		
2 010	109	137	142	144			
2 011	166	229	241				
2 012	193	247					
2 013	161						

Cumulative paid claims

	Development year						
Accident year	0	1	2	3	4	5	6
2 007	n/a	n/a	n/a	673	720	711	711
2 008	n/a	n/a	712	1 057	1 710	1 898	
2 009	n/a	394	586	646	644		
2 010	216	541	703	728			
2 011	275	1 267	1 498				
2 012	205	682					
2 013	100						

The payments of a liability portfolio usually have a long tail, for the simple reason that many claims will be contested before a liability is established and the damage is assessed. Therefore you realize immediately that a simple chain ladder prediction is not suitable in this case.

You decide to do your analysis with a Bornhuetter-Ferguson (BF) method for paid claims.

1. Bornhuetter-Ferguson's method, preparation and estimation

Solutions to a-e are collected in f.

a. Transform the triangle of cumulative paid claims into a triangle of incremental paid claims. Pay special attention to include only the incremental values that you actually know. [10 marks]

b. You realize that the BF method requires a measure of risk exposure. Risk exposure is notoriously difficult to quantify in liability insurance. You do not trust the premiums as they are driven by market conditions.

Propose a proxy measure of risk exposure that you can use. Please make sure that you have comparable information for all accident years, for the measure that you choose. [5 marks]

Solution: Use number of claims reported in accident year (development year 0)

In what follows, "claim rate" means claims divided by risk exposure.
You may assume that no claims will be paid with more than 6 years' delay.

c. Estimate the delay-specific claim rates θ_e^* for $e = 0, \cdots, 6.$ [10 marks]

d. Estimate the overall claim rate θ^* (all delays). [10 marks]

e. Estimate the payment pattern π_e^* expressed in percent of ultimate cost. [10 marks]

f. Compile the results of a-e in the table below. [5 marks]

		Incremental paid claims by Development year e						
Accident year j	Risk exposure	0	1	2	3	4	5	6
2 007								
2 008								
2 009				*Preprinted answer sheet available*				
2 010								
2 011								
2 012								
2 013								
Total								

Claim rates θ_e^*	(overall)							
Incremental π_e^*								
Cumulative $\pi_{\leq e}^*$								

Solution to a-f at end.

2. Bornhuetter-Ferguson's method, prediction

a. Estimate the outstanding claim payments for each accident year. [10 marks]

b. Calculate the estimated ultimate claim cost and the estimated ultimate claim rate of each accident year. [10 marks]

c. Compile the results of a-b in the table below. [10 marks]

					Estimated claim cost (Bornhuetter-Ferguson)		
Accident year j	Risk exposure	Paid claims	$\pi^{*}_{\leq 2013-j}$	Overall claim rate	Outstanding	Ultimate	Ultimate claim rate
Specify formula							
2007							
2008			*Preprinted answer sheet available*				
...							
2012							
2013							
Total							

Solution at end.

3. Discounting

The balance date is 31.12.13. Calculate the discounted value of the outstanding claim payments using a discount rate of 5% per annum. You may assume that all claim payments are made at the end of the year. Compile the results in the table below.

		Outstanding claim payments by future payment year						
Accident year j	Total	2014	2015	2016	2017	2018	2019	2020
2 007								
2 008								
2 009				*Preprinted answer sheet available*				
2 010								
2 011								
2 012								
2 013								
Total								

Discount factor								
Discounted value								

[20 marks]

Solution at end.

4. Other information

a. Explain thoroughly what is meant by the policy attachment conditions "claims incurred basis" and "claims made basis". [15 marks]

"Claims incurred principle": a loss is covered if the event leading to the loss happened during the policy period, even if the claim is lodged after the policy has expired.

"Claims made principle", a loss is only covered if the claim is received while the policy is in force (possibly with a limited time extension after the policy has expired).

b. Does the fact that an insurance is written on "claims made basis", automatically imply that the claim payment liabilities it generates are short-tailed? Give reasons for your answer. [15 marks]

No. Claims can still take a long time to settlement due to legal processes etc.

c. Is the portfolio of questions 1-3 written on "claims incurred basis" or on "claims made basis" ? How can you tell? [15 marks]

The portfolio is written on claims incurred basis since the number pf reported claims increases after the accident year (development year > 0).

5. Generalised linear models

The Bornhuetter-Ferguson predictions of questions 1-2 can be replicated using the framework of a generalized linear model (GLM). Show how.

a. Specify the link function. [5 marks]

Solution: log-link function

b. Specify the covariate structure (explanatory variables). [5 marks]

Solution: A model with constant accident year effect: $\mu_{je} = \beta_e$

c. Specify the probability distribution(s) that can be used. [5 marks]

Solution: gamma or Poisson

d. How would you change the covariate structure if you wanted to model claim inflation along calendar years? [5 marks]

Solution: $\mu_{je} = \beta_e + \lambda \cdot (j + e - 1)$ **(calendar year a quantitative variable)**

e. How would you change the covariate structure if you wanted to model claim inflation along accident years? [5 marks]

Solution: $\mu_{je} = \alpha \cdot (j - 1) + \beta_e$ **(accident year a quantitative variable)**

No calculations are required in this question.

6. Model comparison

Most of our models assume that "$\mathrm{E}(X_{je}) = p_j \theta_j \pi_e$" in some way or other.

a. What is the main difference between the models used to derive the Bornhuetter-Ferguson method and the Chain ladder method? [10 marks]

Solution: BF assumes constant θ_j for all accident years, CL assumes that each θ_j must be estimated separately. Fixed parameters in both.

b. What is the main difference between the Bühlmann-Straub model and the models used to derive the BF method and the CL method? [10 marks]

Solution: BS assumes random θ_j, CL and BF assume fixed parameters.

c. What is the main difference between the Bühlmann-Straub model and the Hesselager-Witting model? [10 marks]

Solution: HW assumes random delay probabilities π_e, BS assume fixed probabilities.

THE END

Name:_____

Preprinted answer sheet for Question 1f

		Incremental paid claims by Development year e						
Accident year j	Risk exposure	0	1	2	3	4	5	6
2 007	57	n/a	n/a	n/a	n/a	47	-9	0
2 008	76	n/a	n/a	n/a	345	653	188	
2 009	114	n/a	n/a	192	60	-2		
2 010	109	216	325	162	25			
2 011	166	275	992	231				
2 012	193	205	477					
2 013	161	100						
Total	876	796	1794	585	430	698	179	0

Claim rates θ_e^*	Overall: 12,2126	1,2655	3,8333	1,5039	1,4381	2,8259	1,3459	0,0000
Incremental π_e^*	Formula θ_e^* / θ^*	0,1036	0,3139	0,1231	0,1178	0,2314	0,1102	0,0000
Cumulative $\pi_{\leq e}^*$	Formula $\sum_{e'=0}^{e} \pi_{e'}^*$	0,1036	0,4175	0,5406	0,6584	0,8898	1,0000	1,0000

Name:_____

Preprinted answer sheet for Question 2c

					Estimated claim cost (Bornhuetter-Ferguson)		
Accident year j	Risk exposure	Paid claims	$\pi^*_{\leq 2013-j}$	Overall claim rate	Outstanding	Ultimate	Ultimate claim rate
Specify formula	p_j	$X_{j,\leq 2012-j}$	**Table lookup**	**Table lookup**	$\overline{X}_{j,>2013-j} = p_j \theta^* \pi^*_{>2013-j}$	$X_{j,\leq 2013-j} + \overline{X}_{j,>2013-j}$	$\frac{X_{j,\leq 2013-j} + \overline{X}_{j,>2013-j}}{p_j}$
2 007	**57**	**711**	**100 %**	**12,21**	**0**	**711**	**12,47**
2 008	**76**	**1898**	**100 %**	**12,21**	**0**	**1 898**	**24,97**
2 009	**114**	**644**	**89 %**	**12,21**	**153**	**797**	**6,99**
2 010	**109**	**728**	**66 %**	**12,21**	**455**	**1 183**	**10,85**
2 011	**166**	**1498**	**54 %**	**12,21**	**931**	**2 429**	**14,63**
2 012	**193**	**682**	**42 %**	**12,21**	**1 373**	**2 055**	**10,65**
2 013	**161**	**100**	**10 %**	**12,21**	**1 762**	**1 862**	**11,57**
Total	**876**	**6261**			**4 675**	**10 936**	**12,48**

Name:_____

Preprinted answer sheet for Question 3

		Outstanding claim payments by future payment year						
Accident year *j*	Total	2014	2015	2016	2017	2018	2019	2020
2 007	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
2 008	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
2 009	**153**	**0**	**0**	**0**	**0**	**0**	**0**	**153**
2 010	**308**	**147**	**0**	**0**	**0**	**0**	**0**	**455**
2 011	**239**	**469**	**223**	**0**	**0**	**0**	**0**	**931**
2 012	**290**	**278**	**545**	**260**	**0**	**0**	**0**	**1 373**
2 013	**617**	**242**	**232**	**455**	**217**	**0**	**0**	**1 762**
Total	**1 608**	**1 135**	**1 000**	**715**	**217**	**0**	**0**	**4 675**

Discount factor	**0,9524**	**0,9070**	**0,8638**	**0,8227**	**0,7835**	**0,7462**	**0,7107**	
Discounted value	**1531**	**1030**	**864**	**588**	**170**	**0**	**0**	**4 183**

General formula $\overline{X}_{j,\text{payment year}-2013} = p_j \theta^* \pi^*_{\text{payment year}-2013}$