



Master in Actuarial Science
Loss Reserving
29-06-2016
Time allowed: 2 hours

Instructions:

1. This paper contains **4** questions and comprises **3** pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.
11. Preprinted answer sheets are available for some of the tables required.

The following table shows cumulative paid claims for accident years 2011-2015 at 31.12.2015.

Cumulative claim payments					
	Development year				
Accident year	0	1	2	3	4
2011	n/a	n/a	267	270	273
2012	n/a	270	285	288	
2013	210	342	369		
2014	234	393			
2015	255				

The premium is shown in the next table.

Accident year	Premium
2011	366
2012	378
2013	444
2014	459
2015	501

1. Preparation

- a. Organise the incremental paid claims in a development triangle. [10 marks]

2. Bornhuetter-Ferguson method (calibration and prediction)

In what follows, claim rate means claims divided by exposure.

- a. Estimate the delay-specific claim rates for delays 0 to 4. [10 marks]
- b. Assume that 5% of ultimate claims are paid at delay 5.
Determine a claim rate for delay 5 that (together with the claim rates estimated for delays 0 to 4) reflects that assumption. [10 marks]
- c. Estimate the overall claim rate per accident year (all delays 0-5). [10 marks]
- d. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]
- e. Estimate the outstanding claim payments for each accident year. [10 marks]
- f. Calculate the estimated ultimate claim cost of each accident year. [10 marks]
- g. Calculate the estimated loss ratio of each accident year.
Loss ratio is defined as ultimate claim cost divided by exposure. [10 marks]
- h. Do you notice anything in the loss ratios that could indicate that the Bornhuetter-Ferguson method is not suitable for this portfolio? [10 marks]

3. Bühlmann-Straub model

The central assumptions of the Bühlmann-Straub model are

$$E(X_{je} | \Theta_j) = p_j b(\Theta_j) \pi_e \text{ and } \text{Var}(X_{je} | \Theta_j) = p_j v(\Theta_j) \pi_e.$$

- Explain (briefly) the meaning of the symbols X_{je} , p_j , Θ_j , and π_e . [10 marks]
- Specify the assumptions of the Bühlmann-Straub model, concerning the stochastic (or non-stochastic) behaviour of X_{je} , p_j , Θ_j and π_e . [10 marks]
- Explain the meaning of the parameters $\beta = E(b(\Theta_j))$, $\lambda = \text{Var}(b(\Theta_j))$ and $\varphi = E(v(\Theta_j))$ in words. [10 marks]

Define $b_j^* = X_{j \leq J-j} / p_j \pi_{j \leq J-j}$ (the “chain ladder estimate”) and $\bar{b}_j = z_j b_j^* + (1 - z_j) \beta$ (the “credibility estimate”). Assume that z_j (the “credibility factor”) is non-random.

- Prove that $E(b_j^* | \Theta_j) = b(\Theta_j)$ and $\text{Var}(b_j^* | \Theta_j) = v(\Theta_j) / p_j \pi_{j \leq J-j}$. [10 marks]
- Prove that $Q(z_j) = E(\bar{b}_j - b(\Theta_j))^2 = z_j^2 \frac{\varphi}{p_j \pi_{j \leq J-j}} + (1 - z_j)^2 \lambda$. [10 marks]
- For a future payment X_{je} , prove that the MSEP of the predictor $\bar{X}_{je} = p_j \bar{b}_j \pi_e$ is $E(\bar{X}_{je} - X_{je})^2 = p_j \pi_e \varphi + (p_j \pi_e)^2 Q(z_j)$ [10 marks]
- Derive the choice of credibility factor z_j that minimises $Q(z_j)$. [10 marks]

4. Stages in the life of a claim

- Explain the meaning of the acronyms RNBS, IBNR and CBNI. Please do not just translate the abbreviations, but explain what they mean. [10 marks]
- Suggest a few pieces of information that – in addition to development triangles – could be useful in estimating the ultimate cost of claims RBNS. [10 marks]
- Explain the meaning of this assertion: “Statistically, CBNI claims behave in the same way as IBNR claims.” [10 marks]
- Explain the meaning of this assertion: “Know your RBNS, then IBNR and CBNI come by themselves.” [10 marks]

END