Master in Actuarial Science

Loss Reserving

30-06-2011

Time allowed: 2 hours

Model solution

Instructions:

1. This paper contains 7 questions and comprises 3 pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all 7 questions.
6. Begin your answer to each of the 7 questions on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.

You are the actuary of a general insurance company and have received the following data showing the number of reported claims on 31.12.2005.

	Reporting delay				
Accident year	0	1	2	3	4
2001	126	69	7	4	1
2002	87	58	8	3	
2003	77	45	8		
2004	79	41			
2005	84				

The exposure is shown in the next table.

Accident year	Exposure
2001	5 630
2002	5 124
2003	4 719
2004	3 898
2005	3 575

You may assume that no claims will reported with a delay of more than four years.

1. <u>Bornhuetter-Ferguson method</u>
   a. Estimate the delay-specific claim frequencies. [10 marks]
   b. Estimate the overall claim frequency per accident year. [10 marks]
   c. Estimate the reporting pattern. [10 marks]
   d. Estimate the outstanding number of claims for each accident year. [10 marks]
   e. Fill the missing cells in the run-off triangle with predictions. [10 marks]

$$\theta_0^* = \frac{126+87+77+79+84}{5630+5124+4719+3898+3575} = 1.97\%$$

$$\theta_1^* = \frac{69+58+45+41}{5630+5124+4719+3898} = 1.10\%$$

a. $$\theta_2^* = \frac{7+8+8}{5630+5124+4719} = 0.15\%$$

$$\theta_3^* = \frac{4+3}{5630+5124} = 0.07\%$$

$$\theta_4^* = \frac{1}{5630} = 0.02\%$$

b. $$\theta^* = 1.97\% + 1.10\% + 0.15\% + 0.07\% + 0.02\% = 3.31\%$$

$\pi_0^* = 1.97\% / 3.31\% = 59.73\%$

$\pi_1^* = 1.10\% / 3.31\% = 33.27\%$

c. $\pi_2^* = 0.15\% / 3.31\% = 4.50\%$

$\pi_3^* = 0.07\% / 3.31\% = 1.97\%$

$\pi_4^* = 0.02\% / 3.31\% = 0.54\%$

d.

Accident year	Exposure	Developed to	pi(cum.)	Theta_star	Outstanding
2001	5630	4	100 %	3,31E-02	0,0
2002	5124	3	99 %	3,31E-02	0,9
2003	4719	2	97 %	3,31E-02	3,9
2004	3898	1	93 %	3,31E-02	9,0
2005	3575	0	60 %	3,31E-02	47,6
Total	22946				61,4

e.

Accident year	0,00	1,00	2,00	3,00	4,00
2001,00	126,00	69,00	7,00	4,00	1,00
2002,00	87,00	58,00	8,00	3,00	0,91
2003,00	77,00	45,00	8,00	3,07	0,84
2004,00	79,00	41,00	5,79	2,54	0,69
2005,00	84,00	39,31	5,31	2,33	0,63

2. Chain ladder method

a. Estimate the development factors. [10 marks]

b. Estimate the reporting pattern. [10 marks]

c. Estimate the overall claim frequency per accident year. [10 marks]

d. Estimate the outstanding number of claims for each accident year. [10 marks]

e. Fill the missing cells in the run-off triangle with predictions. [10 marks]

Cumulative claims reported

Accident year	0	1	2	3	4
2001	126	195	202	206	207
2002	87	145	153	156	
2003	77	122	130		
2004	79	120			
2005	84				

a.

$$\delta_1^* = \frac{195+145+122+120}{126+87+77+79} = 157{,}72\%$$

$$\delta_2^* = \frac{202+153+130}{195+145+122+120} = 104.98\%$$

$$\delta_3^* = \frac{206+156}{202+153} = 101.97\%$$

$$\delta_4^* = \frac{207}{206} = 100.49\%$$

b.

	0	1	2	3	4
delta (incr.)		1,5772	1,0498	1,0197	1,0049
delta (cum.)	100 %	158 %	166 %	169 %	170 %
pi (cum.)	58,94 %	92,96 %	97,59 %	99,52 %	100,00 %
pi (incr.)	58,94 %	34,02 %	4,63 %	1,92 %	0,48 %

c.

Accident year	Exposure	Developed to	Observed	pi(cum.)	Theta_star(j)
2001	5630	4	207	100 %	3,68E-02
2002	5124	3	156	100 %	3,06E-02
2003	4719	2	130	98 %	2,82E-02
2004	3898	1	120	93 %	3,31E-02
2005	3575	0	84	59 %	3,99E-02
Total	22946		697		3,31E-02

d.

Accident year	Exposure	Developed to	Observed	pi(cum.)	Theta_star(j)	Outstanding
2001	5630	4	207	100 %	3,68E-02	0,0
2002	5124	3	156	100 %	3,06E-02	0,8
2003	4719	2	130	98 %	2,82E-02	3,2
2004	3898	1	120	93 %	3,31E-02	9,1
2005	3575	0	84	59 %	3,99E-02	58,5
Total	22946		697		3,31E-02	71,6

e.

Accident year	0	1	2	3	4
2001	126,00	69,00	7,00	4,00	1,00
2002	87,00	58,00	8,00	3,00	0,76
2003	77,00	45,00	8,00	2,56	0,64
2004	79,00	41,00	5,97	2,48	0,62
2005	84,00	48,49	6,60	2,74	0,69

3. Benktander's method

   With claim frequency and reporting pattern from question 1, apply Benktander's method to:

   a. Estimate the outstanding number of claims for each accident year. [20 marks]

Accident year	Exposure	Developed to	Observed	pi(cum.)	Theta_BF	Theta_CL	Credibility z	Theta_bar	Outstanding
2001	5630	4	207	100 %	3,31E-02	3,68E-02	100 %	3,68E-02	0,0
2002	5124	3	156	99 %	3,31E-02	3,06E-02	99 %	3,06E-02	0,8
2003	4719	2	130	97 %	3,31E-02	2,83E-02	97 %	2,84E-02	3,4
2004	3898	1	120	93 %	3,31E-02	3,31E-02	93 %	3,31E-02	9,0
2005	3575	0	84	60 %	3,31E-02	3,93E-02	60 %	3,68E-02	53,0
Total	22946		697						66,2

4. Explain how you can evaluate the uncertainty of predictions by bootstrapping. [20 marks]

Outline

For i=1,…,M

- Generate independent $N_{jd}^{(i)} \sim \text{Poisson}\left(p_j \theta_j^* \pi_d^*\right)$ for j=1,…J, d=0,…,D.
- Calculate pseudo-predictions $\overline{N}_{jd}^{(i)}$ for $j+d>J$ using the chosen method.
- Calculate pseudo-errors, for example $E^{(i)} = \sum_{j+d>J} \left(N_{jd}^{(i)} - \overline{N}_{jd}^{(i)}\right)$

Analyse the distribution of $E^{(i)}$.

Simulating a Poisson random variable with small expected value:

```
incremental_probability = Exp(-lambda)
cumulative_probability = incremental_probability
m = 0

Do While cumulative_probability < uniform
  m = m + 1
  incremental_probability = incremental_probability * lambda / m
  cumulative_probability = cumulative_probability + incremental_probability
Loop

Random_Poisson = m
```

For large expected value one can use the approximation Poisson(lambda)~Normal(lambda,lambda).

5. Exposure measures

   a. Discuss possible exposure measures in different lines of insurance: motor vehicle insurance, workers' compensation insurance, liability insurance. [10 marks]

      Motor vehicle: Number of policies or insured vehicles.
      Workers' compensation: Number of insured workers or labour years
      Liability: difficult to quantify exposure. Turnover may be used.

b. Explain why premium not always is a good measure of exposure. [10 marks]

The premium charged by the insurer for assuming a liability does not necessarily reflect the size of the liability. Premium can be used if rates have been reasonably stable over time, or after a correction for known rate changes.

6. Propose methods for smoothing and/or extending the reporting tail beyond delay 4. [10 marks]

Chain ladder: $\delta_d^* = 1 + \gamma(\delta_{d-1}^* - 1)$ for $d > 4$

Bornhuetter-Ferguson: $\theta_d^* = \gamma\theta_{d-1}^*$ for $d > 4$

7. <u>Outstanding claim categories.</u>
   a. Explain the meaning of the abbreviations RBNS, IBNR and CBNI. [10 marks]
   b. Why are IBNR claims more similar to CBNI claims than to RBNS claims? [10 marks]
   c. Determine the category of each of the following claims on 31.12.2005: [10 marks]
   Claim A: Accident occurred 31.08.05, claim reported 05.01.06.
   Claim B: Accident occurred 05.08.05, claim reported 10.08.05.
   Claim C: Accident occurred 01.02.06, claim reported 15.02.06.

a. RBNS means Reported but not Settled, but in this course it has been used in the sense of Reported. IBNR means Incurred but not Reported, CBNI means Covered but not Incurred.
b. For RBNS claims one has potentially a lot of information: Number of claims, claim characteristics, settlement status, settled amounts, partial payments, case estimates etc. For IBNR and CBNI claims one has no information, except the knowledge that they are reported with a certain frequency and a certain severity distribution, both of which one can estimate statistically.
c. Claim A: IBNR. Claim B: RBNS. Claim C: CBNI.