Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

DESDE 1911

Master in Actuarial Science

Loss Reserving

24-06-2015

Time allowed: 2 hours

Instructions:

1. This paper contains **7** questions and comprises **9** pages including the title page and **5** preprinted answer sheets.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.
11. Preprinted answer sheets are available for some of the tables required.

You are the actuary of a general insurance company and have been asked to analyse the claim cost of its liability insurance portfolio. This is the data you receive from the company:

Accident year	Earned premium
2009	336
2010	414
2011	500
2012	536
2013	558
2014	635

Cumulative paid claims

	Development year					
Accident year	0	1	2	3	4	5
2009	N/A	N/A	150	180	211	223
2010	N/A	96	169	218	242	
2011	16	77	144	204		
2012	15	90	158			
2013	11	80				
2014	16					

1. Do you consider the portfolio to be short-tailed or not? Given you answer, what prediction methods would you recommend or not recommend? [5 marks]

   Answer: Relatively long-tailed, chain-ladder not recommended

2. Convert the triangle of cumulative paid claims to incremental paid claims. [15 marks]

3. Estimating the payment pattern.

   a. Estimate delay-specific claim rates, as in the Bornhuetter-Ferguson method. [15 marks]

   b. A little bird has told you that you can expect some 10% of an accident year's total claims to be paid after more than five years' delay. You may assume that the remaining 10% will be paid in development year 6. Calculate a claim rate for delay 6 that amounts to 10% of the total. [10 marks]

   c. Calculate the sum of the delay-specific claims rates. [5 marks]

   d. Calculate a payment pattern for development years 0-6. [10 marks]

   Answer a-d: see answer sheet

   e. What other information could you ask of the company, to help you to assess the proportion of payments to expect after development year 5? [5 marks]

   Answer: Outstanding case estimates!

Calculate your answers with three decimals.

In exercises 2-3, take care to avoid mistakes that could result from the fact that some payment information is missing.

4. Claim predictions

   a. Predict the outstanding claim cost, the ultimate claim cost and the ultimate claim rate of each accident year, using the Bornhuetter-Ferguson prediction method. [15 marks]

   Answer: see answer sheet

   b. Predict the timing of future payments of each accident year, using the estimated payment pattern. [20 marks]

   Answer: see answer sheet

   c. Predict the remaining outstanding claims at the start and the end of each future payment year. [15 marks]

   Answer: see answer sheet

   Calculate your answers with three decimals.

5. Outstanding claim provision according to Solvency II

   Assume an interest rate of 3%.

   The “best estimate”

   a. Calculate the discounted value of the predicted outstanding claims.
   You can assume that payments are made at the end of each payment year.
   You do not need to specify the best estimate by accident year, just in total.
   [15 marks]

   Answer: see answer sheet

   The “risk margin”

   b. Assume that on 1st January of 2015-2020, the solvency capital requirement

(SCR) is 50% of the outstanding claims that you predicted in exercise 3.c. The upfront cost of providing capital for one year is 6%. Calculate the risk margin as the total discounted cost of providing solvency capital for the years 2015-2020. You do not need to specify the risk margin by accident year. [20 marks]

Answer: see answer sheet

Calculate your answers with three decimals.

6. Modelling by GLM

a. Specify the GLM(s) that will reproduce the predictions of the Bornhuetter-Ferguson method. A GLM is fully specified by its covariate structure, its link function and its probability distribution. [15 marks]

Answer:

Covariate structure: $\mu_{je} = \beta_e$ or equivalently, only development year as classification variable.

Link function: logarithmic, giving multiplicative means.

Probability distribution: Poisson or gamma

The Bornhuetter-Ferguson method may be derived by assuming that the expected claim rate is the same for all accident years. That may not always be a realistic assumption. For instance, if you study the ultimate claim rates in 4.a, you see that mature years have higher-than-average ultimate claim rates. You can easily see that premiums have increased at a faster rate than claims. This indicates that the Bornhuetter-Ferguson method was not the best choice.

At the other end of the modelling spectrum one has the chain ladder method, which may be derived by assuming that each accident year's ultimate claim rate is unique. Some would say that using the chain ladder method amounts to ignoring valuable information.

As a compromise between the extreme assumptions, you could assume that the expected claim rates for accident years 2009-2014 follow a trend function, and try to estimate the trend.

b. Specify a GLM that will allow you to estimate the payment pattern and an exponential trend in the accident year claim rates. [20 marks]

Hint: We have seen how to model a calendar year trend. Just make the necessary change in the quantitative variable.

Answer:

Covariate structure: $\mu_{je} = \beta_e + \gamma(j-1)$ **or equivalently, development year as classification variable and accident year as quantitative variable.**

7. Mention briefly some other methods that you can use to learn from the claim experience, when accident years' claim rates are similar but not identical. You get additional marks for a description of their properties. [15 marks]

Answer:

Mention of Benktander method and Bühlmann-Straub method gives 5 marks. Mention of (correct) properties gives additional marks.

THE END

Name:_____

Answer sheet for Question 2

Incremental paid claims	Development year					
Accident year	0	1	2	3	4	5
2009				30	31	12
2010			73	49	24	
2011	16	61	67	60		
2012	15	75	68			
2013	11	69				
2014	16					

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Answer sheet for Question 3a-d

		Development year *e*							
	Specify formula	0	1	2	3	4	5	6	Total
Delay-specific claim rates θ_e^*	$\theta_e^* = \frac{\sum_j X_{je}}{\sum_j p_j}$	0,026	0,129	0,143	0,111	0,073	0,036	0,058	0,576
Incremental payment proportions π_e^*	$\pi_e^* = \frac{\theta_e^*}{\theta^*}$	0,045	0,223	0,249	0,193	0,127	0,062	0,100	1,000
Cumulative payment proportions $\pi_{\leq e}^*$	$\pi_{\leq e}^* = \sum_{e' \leq e} \pi_{e'}^*$	0,045	0,268	0,518	0,711	0,838	0,900	1,000	

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Answer sheet for Question 4a

Accident year *j*	Earned premium	Cumulative Payments to date	pi(cum.)	Theta	Outstanding payments (prediction)	Ultimate payments (prediction)	Ultimate claim rate (prediction)
Specify formula	p_j	$X_{j,\leq 2014-j}$	$\pi^*_{\leq 2014-j}$	θ^*	$\overline{X}_{j,>2014-j} = p_j \theta^* \pi^*_{>2014-j}$		
2009	336	223	0,900	0,576	19,351	242,351	0,721
2010	414	242	0,838	0,576	38,629	280,629	0,678
2011	500	204	0,711	0,576	83,320	287,320	0,575
2012	536	158	0,518	0,576	148,922	306,922	0,573
2013	558	80	0,268	0,576	235,078	315,078	0,565
2014	635	16	0,045	0,576	349,183	365,183	0,575
Total	2 979	923			874,482	1 797,482	0,603

Name:_____

Answer sheet for Question 4b and 4c

	Payment year						
Accident year	2015	2016	2017	2018	2019	2020	Total
2009	19,351	0,000	0,000	0,000	0,000	0,000	19,351
2010	14,786	23,843	0,000	0,000	0,000	0,000	38,629
2011	36,667	17,857	28,796	0,000	0,000	0,000	83,320
2012	59,603	39,307	19,143	30,869	0,000	0,000	148,922
2013	80,044	62,050	40,920	19,929	32,136	0,000	235,078
2014	81,666	91,090	70,612	46,567	22,679	36,571	349,183
Total payments during payment year (prediction)	292,116	234,146	159,471	97,364	54,815	36,571	874,482
Outstanding payments at start of payment year	874,482	582,366	348,220	188,750	91,385	36,571	
Outstanding payments at end of payment year	582,366	348,220	188,750	91,385	36,571	0,000	

Name:_____

Answer sheet for Question 5a and 5b

	Payment year						
	2015	2016	2017	2018	2019	2020	Total
Predicted payments at 31st December of the payment year	292,116	234,146	159,471	97,364	54,815	36,571	874,482
Discount factor at 31st December 2014	0,971	0,943	0,915	0,888	0,863	0,837	
Discounted value of predicted payments at 31st December 2014	283,608	220,705	145,938	86,507	47,284	30,627	BE=814,669

Outstanding payments on 1st January of the payment year	874,482	582,366	348,220	188,750	91,385	36,571	
Solvency capital required during the year	437,241	291,183	174,110	94,375	45,693	18,285	
Upfront cost of solvency capital provided during the year	26,234	17,471	10,447	5,662	2,742	1,097	63,653
Discount factor at 31st December 2014	1,000	0,971	0,943	0,915	0,888	0,863	
Discounted value of cost of capital at 31st December 2014	26,234	16,962	9,847	5,182	2,436	0,946	RM=61,608