Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

DESDE 1911

Master in Actuarial Science

Loss Reserving

29-06-2012

Time allowed: 2 hours

Solution

Instructions:

1. This paper contains 4 questions and comprises 3 pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.

The following data shows paid claims for the period 2007-2011 at 31.12.2011.

Incremental	Payment delay in years					
Accident year	0	1	2	3	4	5
2007	32	210	136	153	117	
2008	105	121	244	116		
2009	46	152	106			
2010	156	216				
2011	107					

Cumulative	Payment delay in years					
Accident year	0	1	2	3	4	5
2007	32	242	378	531	648	
2008	105	226	470	586		
2009	46	198	304			
2010	156	372				
2011	107					

The premium is shown in the next table.

Accident year	Premium
2007	1312
2008	1390
2009	1510
2010	1762
2011	1977

You may assume that no claims will be paid with a delay of more than five years.

1. Bornhuetter-Ferguson method
   a. Estimate the delay-specific claim ratios for delays 0 to 4. [10 marks]
   b. Assume that claims paid up to year 4 make up only 75% of ultimate claim cost.
      As a convenience, you may assume that the remaining 25% is paid in year 5.
      Calculate the delay-specific claim ratio for development year 5. [10 marks]
   c. Estimate the average overall claim ratio per accident year (all delays). [10 marks]
   d. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]
   e. Estimate the outstanding claim payments for each accident year. [10 marks]

a. Calculation: sum of incremental claims per column / sum of corresponding exposures.

	0	1	2	3	4
Claim ratio	5,6 %	11,7 %	11,5 %	10,0 %	8,9 %

b. Calculation: claim ratio #5 = (25/75) * sum of claim ratios #1-4: 15,9 %

c. Calculation: sum of claim ratios for delays 0-5.

Average	63,6 %

d. Calculation: Results from a-b divided by result in c).

	0	1	2	3	4	5
Cumulative pi(cum.)	8,82 %	27,20 %	45,34 %	60,98 %	75,00 %	100,00 %
Incremental pi(inc.)	8,82 %	18,39 %	18,13 %	15,65 %	14,02 %	25,00 %

e. Calculation: Outstanding = Exposure x Loss ratio x (1-pi(cum.))

Accident year	Exposure	Developed to	Observed	pi(cum.)	Loss ratio	Outstanding
2007	1 312	4	648	75,0 %	63,6 %	209
2008	1 390	3	586	61,0 %	63,6 %	345
2009	1 510	2	304	45,3 %	63,6 %	525
2010	1 762	1	372	27,2 %	63,6 %	816
2011	1 977	0	107	8,8 %	63,6 %	1 147
Total	7 951		2 017			3 042

2. Chain ladder method

a. Estimate the year-on-year development factors for delays 0 to 4. [10 marks]

b. Assume that claims paid up to year 4 make up only 75% of ultimate claim cost. As a convenience, you may assume that the remaining 25% is paid in year 5. Calculate the development factor between development years 4 and 5. [10 marks]

c. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]

d. Estimate the overall claim ratio for each accident year. [10 marks]

e. Estimate the outstanding claim payments for each accident year. [10 marks]

a. Calculation: Sum of cumulative claims per column / sum of corresponding cumulative claims in previous column.

Empirical	0	1	2	3	4
Average		306,19 %	172,97 %	131,72 %	122,03 %

b. Calculation: Development factor 5 = 100/75 = 133.33%.

c. Calculation: Accumulate development factors to end, divide accumulated factors for each delay by ultimate

delta to pi	0	1	2	3	4	5
Devt. factor (incr.)		3,0619	1,7297	1,3172	1,2203	1,3333
Devt. factor (cum.)	100 %	306 %	530 %	698 %	851 %	1135 %
pi (cum.)	8,81 %	26,97 %	46,66 %	61,46 %	75,00 %	100,00 %
pi (incr.)	8,81 %	18,16 %	19,68 %	14,80 %	13,54 %	25,00 %

d. Calculation for d. Observed / (Exposure x pi(cum.))

e. Calculation for e. Calculation: Outstanding = Exposure x Loss ratio x (1-pi(cum.))

Accident year	Exposure	Developed to	Observed	pi(cum.)	d. Loss ratio	e. Outstanding
2007	1 312	4	648	75 %	65,9 %	216
2008	1 390	3	586	61 %	68,6 %	367
2009	1 510	2	304	47 %	43,1 %	348
2010	1 762	1	372	27 %	78,3 %	1 007
2011	1 977	0	107	9 %	61,4 %	1 108
Total	7 951		2 017		63,2 %	3 046

3. Bühlmann-Straub model

a. Describe briefly the assumptions of the Bühlmann-Straub model for claim amounts and explain the meaning of the parameters β, φ and λ. [20 marks]

The optimal credibility estimator of the random claim level of accident year *j* has the form $\overline{b}_j = \zeta_j \hat{b}_j + (1-\zeta_j)\beta$, where $\hat{b}_j$ is the chain ladder estimator, β is the prior mean, and ζ_j is the optimal credibility factor.

b. Specify the formula for the optimal credibility factor ζ_j. [10 marks]

c. Explain in what way the Bornhuetter-Ferguson method and the chain ladder method can be seen as limiting cases of the Bühlmann-Straub credibility method. [10 marks]

The following parameter values in a Bühlmann-Straub model have been estimated:

β (beta)	0.63
φ (phi)	8
λ (lambda)	0.003

d. Use these parameter values and the payment pattern $\{\pi_e, e = 0,1,\cdots\}$ from problem 2 to complete the following table:

Accident year	Exposure	Observed	$\pi_{\leq J-j}$	$\hat{b}_j$	β	ζ_j	$\overline{b}_j$	Outstanding
2007	1 312	648						
2008	1 390	586						
2009	1 510	304						
2010	1 762	372						
2011	1 977	107						
Total	7 951	2 017						

[20 marks]

a. The candidate should mention:

- Conditional on an unobserved risk parameter that we denote by Θ_j, the incremental payments $X_{j0}, X_{j1}, \cdots$ are stochastically independent with conditional mean $\mathrm{E}(X_{je} \mid \Theta_j) = p_j b(\Theta_j)\pi_e$ and variance $\mathrm{Var}(X_{je} \mid \Theta_j) = p_j v(\Theta_j)\pi_e$.
- The unobserved risk parameter Θ_j is seen as the outcome of a random variable.
- The $\Theta_1, \cdots, \Theta_J$ are stochastically independent and identically distributed. We denote the mean and variance of the function $b(\Theta_j)$ by $\beta = \mathrm{E}(b(\Theta_j))$ and $\lambda = \mathrm{Var}(b(\Theta_j))$.
- We denote the mean of the function $v(\Theta_j)$ by $\varphi = \mathrm{E}(v(\Theta_j))$.

b. Optimal credibility factor $\zeta_j = \dfrac{\lambda p_j \pi_{\leq J-j}}{\lambda p_j \pi_{\leq J-j} + \varphi}$

c. Bornhuetter-Ferguson is limiting case for $\lambda \to 0$.
Chain ladder is a limiting case for $\lambda \to \infty$ or $\varphi \to 0$

d. Calculation:

$\pi_{\leq J-j}$ = cumulative payment proportion from chain ladder method (1.c)

$\hat{b}_j$ = Observed / (Exposure x $\pi_{\leq J-j}$)

β = given

$\zeta_j = \dfrac{\lambda p_j \pi_{\leq J-j}}{\lambda p_j \pi_{\leq J-j} + \varphi}$ as specified in 3.b

$\bar{b}_j = \zeta_j \hat{b}_j + (1-\zeta_j)\beta$ as specified in text leading up to 3.b

Outstanding = $p_j \bar{b}_j (1 - \pi_{\leq J-j})$

Accident year	Exposure	Observed	$\pi_{\leq J-j}$	$\hat{b}_j$	β	ζ_j	$\bar{b}_j$	Outstanding
2007	1 312	648	75,0 %	65,9 %	63,0 %	27,0 %	63,8 %	209
2008	1 390	586	61,5 %	68,6 %	63,0 %	24,3 %	64,4 %	345
2009	1 510	304	46,7 %	43,1 %	63,0 %	20,9 %	58,9 %	474
2010	1 762	372	27,0 %	78,3 %	63,0 %	15,1 %	65,3 %	840
2011	1 977	107	8,8 %	61,4 %	63,0 %	6,1 %	62,9 %	1 134
Total	7 951	2 017						3 002

4. Choice of method
   a. Would you describe the payment pattern as short-tailed or long-tailed? [10 marks]
   b. Explain the properties of the Bornhuetter-Ferguson method, the chain ladder method and the Bühlmann-Straub method (robustness, sensitivity). [10 marks]
   c. Which method would you choose for the portfolio shown here, and why? [10 marks]
   d. Name at least two ways of assessing the uncertainty of the estimates. [10 marks]

a. Long-tailed (assuming 5 years development was for convenience only).

b. Chain ladder sensitive, responsive to actual experience.
Bornhuetter-Ferguson less sensitive, less responsive to actual experience.
Bühlmann-Straub a compromise.

c. Use Bornhuetter-Ferguson or Bühlmann-Straub, not Chain ladder. Too little accumulated experience in early development years to get reliable projections from chain ladder.

d. The candidate should mention
- **Calculating mean squared error in the BS model.**
- **Bootstrapping.**

Points will also be given for "Mack's model".