Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

DESDE 1911

Master in Actuarial Science

Loss Reserving

17-06-2014

Time allowed: 2 hours

Instructions:

1. This paper contains **5** questions and comprises **8** pages including the title page and **3** preprinted answer sheets.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.
11. Preprinted answer sheets are available for some of the tables required.

You have been asked to estimate the ultimate claim cost of a portfolio of long-tailed insurance that has been running for four years only. The company has given you its premiums and paid claims.

Company Statistics

Accident year	Premium M€	Paid claims M€
2010	63	5
2011	99	11
2012	155	12
2013	178	5

A quick glance convinces you that the company's statistics alone are useless for estimating the ultimate claims, given the long-tailed nature of the business.

Fortunately you have access to seven years of industry statistics, shown below.

Industry Statistics

		Accumulated paid claims by Development year (M€)						
Accident year	Premium M€	0	1	2	3	4	5	6
2006	2 329	33	139	265	464	661	847	949
2007	2 495	39	163	319	503	709	869	
2008	2 649	60	186	368	574	813		
2009	2 674	65	193	348	514			
2010	2 584	31	129	256				
2011	2 561	42	164					
2012	2 526	30						
Sum	17 817	300	974	1 556	2 055	2 183	1 716	949

1. Estimating a payment pattern from the Industry Statistics

Solutions to a-c are collected in d.

a. Estimate year-on-year development factors δ_e^* for $e = 1, \cdots, 6.$ [10 marks]

b. Assume that claims paid until development year 6 make up only 2/3 (66.7%) of ultimate claim cost. Calculate the development factor that applies to the development from development year 6 to ultimate. For simplicity you may denote it by δ_7^*.

[10 marks]

c. Transform the set of estimated development factors $\delta_1^*,\cdots,\delta_7^*$ into a payment pattern $\pi_0^*,\cdots,\pi_7^*$, where π_e^* denotes the expected proportion of ultimate claim payments, that will be made in development year *e*. [10 marks]

d. Collect the results of a-c above in this table.

	Specify formula	0	1	,,,	6	7 to ultimate
δ_e^*						
Δ_e^*		*Preprinted answer sheet available*				
$\pi_{\leq e}^*$						
π_e^*						

The meaning of the symbols is:

δ_e^* Year-on-year development factor,

Δ_e^* Accumulated development factor,

$\pi_{\leq e}^*$ Accumulated payment proportion (current-to-ultimate),

π_e^* Incremental payment proportion (payment year to ultimate).

Please display your results as ratios with four decimals (x.xxxx) or as percentages with two decimals (x.xx%). [5 marks]

Solution to a-d at end.

e. Estimate the ultimate claim cost of the Industry Portfolio using the Chain ladder method (CL). Display your results in the table below. [10 marks]

Accident year *j*	Premium	Paid claims	$\pi_{\leq 2012-j}^*$	Estimated claim rate (CL)	Estimated Outstanding claim cost (CL)	Estimated Ultimate claim cost (CL)
Specify formula						
2006						
2007		*Preprinted answer sheet available*				
...						
2011						
2012						
Total				(average)		

f. Use the Cape Cod (CC) method to calculate the average claim rate. [10 marks]

Solution to e-f at end.

2. Estimating the claim cost of the company

Now you get to the task that your actually were asked to do:
to estimate the ultimate claim cost of the company portfolio.

a. Estimate the ultimate claim cost of the Company Portfolio using Benktander's method. Display your results in the table below.

				Estimated claim rate			Estimated claim cost (Benktander)		
Accident year j	Premium	Paid claims	$\pi^*_{\leq 2013-j}$	Chain ladder	Average	Benktander	Outstanding	Ultimate	Ultimate claim rate
Specify formula									
2010			*Preprinted answer sheet available*						
...									
2013									
Total				(average)					

[25 marks]

Solution at end.

b. The company is convinced that its own portfolio is more profitable than the industry portfolio. Just taking the numbers at face value, and leaving aside all doubts about the paucity of the company data or the relevance of the industry's payment pattern, do your results support or contradict the company's assertion?

[10 marks]

Solution: Industry average claim rate = 0.59, company 0.51, so the results do not contradict the company's assertion.

c. A board member says to you "*I've never heard of this Ben Thunder. Every actuary I know uses the chain ladder. Why aren't you using it?*"

Explain why, in your opinion, using the Chain ladder method for the Company Portfolio would not be a very good idea. Use the actual numbers you have, to illustrate your point.

[15 marks]

Solution: The volatility of its predictions make the CL unsuitable. It should not be used before the current-to-ultimate ratios are at least, say 40-50%. In this portfolio and according to the industry statistics, that takes about 4 development years, cf. question 1 d. The table in question 2 a shows the claim rates that would result from using the CL, and they vary between 24% and 92%. The claim rates from the Benktander method are stable for the last 3 years. But the Benktander method is not as insensitive as Bornhuetter-Ferguson's method.

3. Explaining mechanisms of claim development.

Use as context that the insurance generating the numbers above is for occupational injury and diseases, often called "Workers' Compensation insurance".

a. Explain the meaning of the acronym RBNS, and give three possible reasons why the assessed cost of a reported claim can change before it is settled. [10 marks]

Solution: RBNS means Reported but not settled, claims that the company knows about but where it expects further payments. Possible reasons: New medical reports, court verdicts, new court verdicts on similar claims, deterioration or improvement of the claimant's condition, inflation. Or any other reason that is relevant in the context of WC insurance.

b. Explain the meaning of the acronym IBNR, and give three possible reasons why claim notification can be delayed. [10 marks]

Solution: Incurred but not reported. The loss leading to the right to claim has happened but no claim has been made yet, or at least no claim has been filed. Possible reasons: Gradual deterioration of the injured's health, lack of awareness of right to claim, initial reliance on social security. Or any other reason that is relevant in the context of WC insurance.

c. Explain thoroughly the meaning of the acronym CBNI. [10 marks]

Solution: Covered but not incurred. Claims that will arise from future losses that are covered under in-force policies. On the balance date they are considered a liability. Not to be confused with claims that arise from policies that will be written/renewed after the balance date.

d. In what sense are claims IBNR similar to claims CBNI? [10 marks]

Solution: Both IBNR and CBNI are claims that will be reported only after the balance date, thus unknown in number and nature. Both can be estimated by a simple "risk exposure x claim frequency x average severity" calculation.

4. Explain how claim development and outstanding claim prediction can be cast into the mould of Generalised Linear Models (GLM).

a. What link function would you use? [5 marks]

Solution: In loss reserving we usually use the log-link function, which results in multiplicative means.

b. Give three examples of sensible covariate structures. [15 marks]

Solution, for example:
A model with multiplicative means: $\mu_{je} = \alpha_j + \beta_e$
A model with constant accident year effect: $\mu_{je} = \alpha + \beta_e$
A model with inflation at a constant rate: $\mu_{je} = \alpha + \beta_e + \lambda \cdot (j + e - 1)$

c. What probability distribution would you use and why? [10 marks]

Solution: Gamma for claim amounts, Poisson for claim counts. The distribution should be appropriate for the phenomenon it is describing.

d. Given parameter estimates, how do you calculate predictions? [5 marks]

Solution: $\hat{X}_{je} = p_j \exp\left(\mathbf{m}'_{je} \cdot \hat{\boldsymbol{\beta}}\right)$ for $j + e > J$

where

X_{je} **are future incremental payments and** $\hat{X}_{je}$ **their predictions,**

p_j **are measures of risk exposure,**

$\mathbf{m}'_{je}$ **are covariate vectors, and**

$\hat{\boldsymbol{\beta}}$ **are the parameter estimates.**

Please specify precisely the meaning of any symbols you use.
It is not sufficient to write up a formula without telling what its symbols express.

5. Bühlmann-Straub model

a. Describe briefly the assumptions of the Bühlmann-Straub model for claim amounts and explain the meaning of its parameters β, φ and λ.

[15 marks]

Solution:

The candidate should mention:

- **Conditional on an unobserved risk parameter that we denote by Θ_j, the incremental payments $X_{j0}, X_{j1}, \cdots$ are stochastically independent with conditional mean $\mathrm{E}(X_{je} \mid \Theta_j) = p_j b(\Theta_j)\pi_e$ and variance $\mathrm{Var}(X_{je} \mid \Theta_j) = p_j v(\Theta_j)\pi_e$.**
- **The unobserved risk parameter Θ_j is seen as the outcome of a random variable.**
- **The $\Theta_1, \cdots, \Theta_J$ are stochastically independent and identically distributed. We denote the mean and variance of the function $b(\Theta_j)$ by $\beta = \mathrm{E}(b(\Theta_j))$ and $\lambda = \mathrm{Var}(b(\Theta_j))$.**
- **We denote the mean of the function $v(\Theta_j)$ by $\varphi = \mathrm{E}(v(\Theta_j))$.**
- **Optimal credibility factor $\zeta_j = \dfrac{\lambda p_j \pi_{\leq J-j}}{\lambda p_j \pi_{\leq J-j} + \varphi}$**

b. Explain under what conditions the chain ladder method can be derived from the Bühlmann-Straub model.

[5 marks]

Solution: Chain ladder will be optimal if $\lambda \to \infty$ or $\varphi \to 0$

THE END

Solution for Question 1 a-d

		Development year *e*							
	Specify formula	0	1	2	3	4	5	6	7 to ultimate
δ_e^*	$\delta_e^* = \sum_{j=1}^{J-e} X_{j,\leq e} / \sum_{j=1}^{J-e} X_{j,\leq e-1}$		360,74 %	192,10 %	158,08 %	141,66 %	125,26 %	112,04 %	150,00 %
Δ_e^*	$\Delta_e^* = \prod_{e'=0}^{e} \delta_{e'}^*$	100,00 %	360,74 %	692,98 %	1095,44 %	1551,81 %	1943,73 %	2177,80 %	3266,71 %
$\pi_{\leq e}^*$	$\pi_{\leq e}^* = \Delta_e^* / \Delta_7^*$	3,06 %	11,04 %	21,21 %	33,53 %	47,50 %	59,50 %	66,67 %	100,00 %
π_e^*	$\pi_e^* = \pi_{\leq e}^* - \pi_{\leq e-1}^*$	3,06 %	7,98 %	10,17 %	12,32 %	13,97 %	12,00 %	7,17 %	33,33 %

Solution for Question 1e-f

Accident year j	Premium	Paid claims	$\pi^*_{\leq 2012-j}$	Estimated claim rate (CL)	Estimated Outstanding claim cost (CL)	Estimated Ultimate claim cost (CL)
Specify formula	p_j	$X_{j,\leq 2012-j}$	Table lookup	$\theta^*_j = \frac{X_{j,\leq 2012-j}}{p_j \pi^*_{\leq 2012-j}}$	$\overline{X}_{j,>2012-j} = p_j \theta^*_j \left(1-\pi^*_{\leq 2012-j}\right)$	$X_{j,\leq 2012-j} + \overline{X}_{j,>2012-j}$
2006	2 329	949	67 %	0,61	475	1 424
2007	2 495	869	60 %	0,59	591	1 460
2008	2 649	813	48 %	0,65	898	1 711
2009	2 674	514	34 %	0,57	1 019	1 533
2010	2 584	256	21 %	0,47	951	1 207
2011	2 561	164	11 %	0,58	1 321	1 485
2012	2 526	30	3 %	0,39	950	980
Total	17 817	3 595		Average 0,59	6 205	9 800

Average by Cape Cod method:

$$\theta^* = \sum_{j=2006}^{2012} X_{j,\leq 2012-j} \Big/ \sum_{j=2006}^{2012} p_j \pi^*_{\leq 2012-j} = \sum_{j=2006}^{2012} p_j \pi^*_{\leq 2012-j} \theta^*_j \Big/ \sum_{j=2006}^{2012} p_j \pi^*_{\leq 2012-j}$$

Solution for Question 2a

				Estimated claim rate			Estimated claim cost (Benktander)		
Accident year *j*	**Premium**	**Paid claims**	$\pi^*_{\leq 2013-j}$	**Chain ladder**	**Average**	**Benktander**	**Outstanding**	**Ultimate**	**Ultimate claim rate**
Specify formula	p_j	$X_{j,\leq 2013-j}$	**Table lookup**	$\theta^*_j = \frac{X_{j,\leq 2013-j}}{p_j \pi^*_{\leq 2013-j}}$	θ^*	$\bar{\theta}_j = \frac{\pi^*_{\leq 2013-j}\theta^*_j + \pi^*_{>2013-j}\theta^*}{\pi^*_{\leq 2013-j}}$	$\bar{X}_{j,>2013-j} = p_j \bar{\theta}_j \pi^*_{>2013-j}$	$X_{j,\leq 2013-j} + \bar{X}_{j,>2013-j}$	$\frac{X_{j,\leq 2013-j} + \bar{X}_{j,>2013-j}}{p_j}$
2010	**63**	**5**	**34 %**	**0,24**	**0,51**	**0,42**	**18**	**23**	**0,36**
2011	**99**	**11**	**21 %**	**0,52**	**0,51**	**0,51**	**40**	**51**	**0,52**
2012	**155**	**12**	**11 %**	**0,70**	**0,51**	**0,53**	**73**	**85**	**0,55**
2013	**178**	**5**	**3 %**	**0,92**	**0,51**	**0,52**	**90**	**95**	**0,53**
Total	**495**	**33**		**Average 0,51**			**221**	**254**	**0,51**

Average by Cape Cod method:

$$\theta^* = \sum_{j=2010}^{2013} X_{j,\leq 2013-j} \Big/ \sum_{j=2010}^{2013} p_j \pi^*_{\leq 2013-j} = \sum_{j=2010}^{2013} p_j \pi^*_{\leq 2013-j} \theta^*_j \Big/ \sum_{j=2010}^{2013} p_j \pi^*_{\leq 2013-j}$$