Instituto Superior de Economia e Gestão
UNIVERSIDADE TÉCNICA DE LISBOA
DESDE 1911

Master in Actuarial Science
Loss Reserving
29-06-2016
Time allowed: 2 hours

SOLUTION

Instructions:

1. This paper contains **4** questions and comprises **3** pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.
11. Preprinted answer sheets are available for some of the tables required.

The following table shows cumulative paid claims for accident years 2011-2015 at 31.12.2015.

Cumulative claim payments

	Development year				
Accident year	0	1	2	3	4
2011	n/a	n/a	267	270	273
2012	n/a	270	285	288	
2013	210	342	369		
2014	234	393			
2015	255				

The premium is shown in the next table.

Accident year	Premium
2011	366
2012	378
2013	444
2014	459
2015	501

1. Preparation

   a. Organise the incremental paid claims in a development triangle. [10 marks]

Incremental claim payments

	Development year				
Accident year	**0**	**1**	**2**	**3**	**4**
2011				**3**	**3**
2012			**15**	**3**	
2013	**210**	**132**	**27**		
2014	**234**	**159**			
2015	**255**				

2. Bornhuetter-Ferguson method (calibration and prediction)

   In what follows, claim rate means claims divided by exposure.

   a. Estimate the delay-specific claim rates for delays 0 to 4. [10 marks]

Formula: sum incremental payments in a column / sum of corresponding premiums

	Development year				
Empirical	**0**	**1**	**2**	**3**	**4**
Average	**49.8 %**	**32.2 %**	**5.1 %**	**0.8 %**	**0.8 %**

   b. Assume that 5% of ultimate claims are paid at delay 5.
   Determine a claim rate for delay 5 that (together with the claim rates estimated for delays 0 to 4) reflects that assumption. [10 marks]

Formula: claim rate 5 = sum (claim rates 0-4) * 0.05/0.95 = 4.7%

c. Estimate the overall claim rate per accident year (all delays 0-5). [10 marks]

Formula: overall claim rate = sum (claim rates 0-5) = 93.4%

d. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]

Formula: incremental payment proportion = delay-specific claim rate / overall claim rate

	Development year					
Payment pattern	0	1	2	3	4	5
pi (cum.)	53.29 %	87.79 %	93.26 %	94.12 %	95.00 %	100.00 %
pi (incr.)	53.29 %	34.50 %	5.47 %	0.86 %	0.88 %	5.00 %

e. Estimate the outstanding claim payments for each accident year. [10 marks]

f. Calculate the estimated ultimate claim cost of each accident year. [10 marks]

g. Calculate the estimated loss ratio of each accident year.
Loss ratio is defined as ultimate claim cost divided by exposure. [10 marks]

Accident year	Premium	Observed	pi(cum.)	Claim rate	e Outstanding Premium x claim rate x (1-pi(cum.))	f Ultimate Observed + Outstanding	g Loss ratio Ultimate / Premium
2011	366	273	95.0 %	93.4 %	17.1	290	79.3 %
2012	378	288	94.1 %	93.4 %	20.8	309	81.7 %
2013	444	369	93.3 %	93.4 %	28.0	397	89.4 %
2014	459	393	87.8 %	93.4 %	52.4	445	97.0 %
2015	501	255	53.3 %	93.4 %	218.6	474	94.5 %
Total	2 148	1 578			336.8	1 915	89.1 %

h. Do you notice anything in the loss ratios that could indicate that the Bornhuetter-Ferguson method is not suitable for this portfolio? [10 marks]

Increasing loss ratio over time indicates that it's not appropriate to assume the same average claim rate of 93.4% for all accident years.

3. Bühlmann-Straub model

The central assumptions of the Bühlmann-Straub model are
$\mathrm{E}(X_{je} \mid \Theta_j) = p_j b(\Theta_j)\pi_e$ and $\mathrm{Var}(X_{je} \mid \Theta_j) = p_j v(\Theta_j)\pi_e$.

a. Explain (briefly) the meaning of the symbols X_{je}, p_j, Θ_j, and π_e. [10 marks]

X_{je} **developing claims (payments or counts),** p_j **exposure (however defined),** Θ_j **variable risk conditions (latent),** π_e **development pattern in percent of ultimate.**

b. Specify the assumptions of the Bühlmann-Straub model, concerning the stochastic (or non-stochastic) behaviour of X_{je}, p_j, Θ_j and π_e. [10 marks]

X_{je} **conditionally independent given** Θ_j **,**

p_j **nonrandom, fixed and known,**

Θ_j **not observed and i.i.d,**

π_e **nonrandom, fixed and known.**

c. Explain the meaning of the parameters $\beta = \mathrm{E}(b(\Theta_j))$, $\lambda = \mathrm{Var}(b(\Theta_j))$ and $\varphi = \mathrm{E}(v(\Theta_j))$ in words. [10 marks]

$\beta = \mathrm{E}(b(\Theta_j))$ **expected level of claim rate, à priori,**

$\lambda = \mathrm{Var}(b(\Theta_j))$ **variability/uncertainty in claim rate between accident years (inter-year),**

$\varphi = \mathrm{E}(v(\Theta_j))$ **variability of claims around a given level (intra-year).**

Define $b_j^* = X_{j,\le J-j} / p_j \pi_{\le J-j}$ (the "chain ladder estimate") and $\overline{b}_j = z_j b_j^* + (1-z_j)\beta$ (the "credibility estimate"). Assume that z_j (the "credibility factor") is non-random.

d. Prove that $\mathrm{E}(b_j^* \mid \Theta_j) = b(\Theta_j)$ and $\mathrm{Var}(b_j^* \mid \Theta_j) = v(\Theta_j) / p_j \pi_{\le J-j}$. [10 marks]

Solution (by conditional independence of X_{je} **, given** Θ_j

$$\mathrm{E}(b_j^* \mid \Theta_j) = \mathrm{E}\left(\frac{X_{j,\le J-j}}{p_j \pi_{\le J-j}} \mid \Theta_j\right) = \mathrm{E}\left(\frac{p_j b(\Theta_j)\pi_{\le J-j}}{p_j \pi_{\le J-j}} \mid \Theta_j\right) = b(\Theta_j)$$

$$\mathrm{Var}(b_j^* \mid \Theta_j) = \mathrm{Var}\left(\frac{X_{j,\le J-j}}{p_j \pi_{\le J-j}} \mid \Theta_j\right) = \frac{p_j v(\Theta_j)\pi_{\le J-j}}{(p_j \pi_{\le J-j})^2} = \frac{v(\Theta_j)}{p_j \pi_{\le J-j}}$$

e. Prove that $\mathrm{Q}(z_j) = \mathrm{E}(\overline{b}_j - b(\Theta_j))^2 = z_j^2 \dfrac{\varphi}{p_j \pi_{\le J-j}} + (1-z_j)^2 \lambda$. [10 marks]

Solution

$$\mathrm{Q}(z_j) = \mathrm{E}(\overline{b}_j - b_j)^2 = \mathrm{E}(z_j b_j^* + (1-z_j)\beta - b_j)^2 = \mathrm{E}(z_j(b_j^* - b_j) + (1-z_j)(\beta - b_j))^2$$

$$= z_j^2 \mathrm{E}(b_j^* - b_j)^2 + (1-z_j)^2 \mathrm{E}(\beta - b_j)^2 + 2z_j(1-z_j)\mathrm{E}((b_j^* - b_j)(\beta - b_j))$$

$$= z_j^2 \mathrm{EE}((b_j^* - b_j)^2 \mid \Theta_j) + (1-z_j)^2 \mathrm{E}(\beta - b_j)^2 + 2z_j(1-z_j)\mathrm{EE}(((b_j^* - b_j)(\beta - b_j)) \mid \Theta_j)$$

$$= z_j^2 \mathrm{E}\mathrm{Var}(b_j^* \mid \Theta_j) + (1-z_j)^2 \mathrm{Var}(b_j) + 0 = z_j^2 \frac{\varphi}{p_j \pi_{\le J-j}} + (1-z_j)^2 \lambda$$

f. For a future payment X_{je}, prove that the MSEP of the predictor $\overline{X}_{je} = p_j \overline{b}_j \pi_e$ is

$$\mathrm{E}\left(\overline{X}_{je} - X_{je}\right)^2 = p_j \pi_e \varphi + \left(p_j \pi_e\right)^2 \mathrm{Q}(z_j)$$ [10 marks]

Solution

$$\mathrm{E}\left(\overline{X}_{je} - X_{je}\right)^2 = \mathrm{E}\left(p_j \overline{b}_j \pi_e - X_{je}\right)^2 = \mathrm{E}\left(\left(p_j \overline{b}_j \pi_e - p_j b_j \pi_e\right) - \left(X_{je} - p_j b_j \pi_e\right)\right)^2 =$$
$$\mathrm{E}\left(p_j \overline{b}_j \pi_e - p_j b_j \pi_e\right)^2 + \mathrm{E}\left(X_{je} - p_j b_j \pi_e\right)^2 - 2\mathrm{E}\left(\left(p_j \overline{b}_j \pi_e - p_j b_j \pi_e\right)\left(X_{je} - p_j b_j \pi_e\right)\right) =$$
$$\left(p_j \pi_e\right)^2 \mathrm{E}\left(\overline{b}_j - b_j\right)^2 + \mathrm{EE}\left(\left(X_{je} - p_j b_j \pi_e\right)^2 \mid \Theta_j\right) - 2\mathrm{EE}\left(\left(\left(p_j \overline{b}_j \pi_e - p_j b_j \pi_e\right)\left(X_{je} - p_j b_j \pi_e\right)\right) \mid \Theta_j\right) =$$
$$\left(p_j \pi_e\right)^2 \mathrm{Q}\left(z_j\right) + \mathrm{EVar}\left(X_{je} \mid \Theta_j\right) - 0 \text{ (because past cond. indep. of future, given } \Theta_j) =$$
$$\left(p_j \pi_e\right)^2 \mathrm{Q}\left(z_j\right) + p_j \pi_e \varphi.$$

g. Derive the choice of credibility factor z_j that minimises $\mathrm{Q}(z_j)$. [10 marks]

Solution

Solve $\frac{\partial}{\partial z_j}\mathrm{Q}(z_j) = 2z_j \frac{\varphi}{p_j \pi_{\le J-j}} - 2\left(1 - z_j\right)\lambda = 0$ **to obtain** $\zeta_j = \frac{p_j \pi_{\le J-j} \lambda}{p_j \pi_{\le J-j} \lambda + \varphi}$.

4. Stages in the life of a claim

a. Explain the meaning of the acronyms RNBS, IBNR and CBNI. Please do not just translate the abbreviations, but explain what they mean. [10 marks]

RBNS – reported but not settled, claim not fully paid, may be reopened, etc
IBNR – incurred but not reported, the loss has occurred but the insurer not notified etc
CBNI – covered but not incurred, claims from future losses under existing contracts etc

b. Suggest a few pieces of information that – in addition to development triangles – could be useful in estimating the ultimate cost of claims RBNS. [10 marks]

Type of loss, severity of damage/injury, pre-injury income, previous payments, case estimates etc.

c. Explain the meaning of this assertion:
"Statistically, CBNI claims behave in the same way as IBNR claims." [10 marks]

Both are unreported, we know neither their number nor their nature, but statistically they occur with a certain regularity, depending on exposure, claim frequency, reporting pattern

d. Explain the meaning of this assertion:
"Know your RBNS, then IBNR and CBNI come by themselves." [10 marks]

Claims RBNS is what we commonly call "data". We must analyse the data to get an idea of claim frequencies and claim severities. When we have those, estimating IBNR/CBNI is (relatively) easy.

END