



Master in Actuarial Science
Loss Reserving
16-06-2015
Time allowed: 2 hours

Instructions:

1. This paper contains **5** questions and comprises **8** pages including the title page and **4** preprinted answer sheets.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.
11. Preprinted answer sheets are available for some of the tables required.

You are the actuary of the St. Thomas Mutual Assurance and have been asked to analyse the claim cost of its home and contents insurance portfolio. This is the data you receive from the company:

| Accident year | Earned premium |
|---------------|----------------|
| 2010 | 153 |
| 2011 | 167 |
| 2012 | 191 |
| 2013 | 206 |
| 2014 | 219 |

Cumulative paid claims

| | Development year | | | | |
|---------------|------------------|-----|-----|-----|-----|
| Accident year | 0 | 1 | 2 | 3 | 4 |
| 2010 | 78 | 131 | 135 | 139 | 140 |
| 2011 | 85 | 144 | 154 | 159 | |
| 2012 | 80 | 143 | 152 | | |
| 2013 | 86 | 140 | | | |
| 2014 | 93 | | | | |
| Sum | 422 | 558 | 441 | 298 | 140 |

1. Give a brief definition of the following terms:

- a. Estimate [2 marks]
- b. Provision [2 marks]
- c. Premium provision [2 marks]
- d. Outstanding claim provision [2 marks]
- e. Reserve [2 marks]

Estimate/prediction: an attempt to quantify an uncertain future cost or income. Made by the actuary.

Provision: An item in the balance sheet that is set aside to finance an estimated future cost. Made by the accountant.

Premium provision: Provision for claims covered but not yet incurred on the balance date.

Outstanding claim provision: Provision for claims already incurred on the balance date.

Reserve: An extra, often declared as a provision but deemed to be in excess of the expected cost. Made by management.

2. Payment pattern

You may assume that all claims are settled after a delay of four years.

- a. Estimate the payment pattern using development factors. [15 marks]

See answer sheet

- b. Would you characterize the portfolio as long-tailed or short-tailed? [5 marks]

Short-tailed, as about 50% of all payments in the accident year and little payments left after 3 years.

Calculate your answers with three decimals.

3. Claim predictions

- a. Predict the outstanding claim cost and the ultimate claim cost of each accident year, using the chain ladder prediction method. [15 marks]

See answer sheet

- b. Predict the timing of future payments of each accident year, using the estimated payment pattern. [20 marks]

See answer sheet

- c. Predict the remaining outstanding claims at the start and the end of each future payment year. [15 marks]

See answer sheet

Calculate your answers with three decimals.

4. Outstanding claim provision according to Solvency II

Assume an interest rate of 5%.

The “best estimate”

- a. Calculate the discounted value of the predicted outstanding claims.
You can assume that payments are made at the end of each payment year.
You do not need to specify the best estimate by accident year, just in total.

[15 marks]

See answer sheet

The “risk margin”

- b. Explain the cost-of-capital method for calculating a risk margin.

[20 marks]

The student should mention (4 marks each):

- **The idea of another company taking over the liability**
- **The need for solvency capital to support the liability**
- **That solvency capital is needed until all claims are settled**
- **The notional charge for providing solvency capital every year**
- **That the charge is upfront (affects the discount factor)**

- c. Assume that on 1st January of 2015, 2016, 2017, 2018, the solvency capital requirement (SCR) is 50% of the outstanding claims that you predicted in exercise 3.c. The upfront cost of providing capital for one year is 6%. Calculate the risk margin as the total discounted cost of providing solvency capital for the years 2015-2018. You do not need to specify the risk margin by accident year.

[20 marks]

See answer sheet

The outstanding claim provision according to Solvency II is the sum of best estimate and risk margin.

Calculate your answers with three decimals.

5. Modelling by GLM

The predictions of the chain ladder method can be reproduced within a generalised linear model (GLM).

- a. Specify the GLM that will reproduce the predictions of the chain ladder method. A GLM is fully specified by its covariate structure, its link function and its probability distribution. [15 marks]

Covariate structure: $\mu_{je} = \alpha_j + \beta_e$ **or equivalently, accident year and development year as classification variables.**

Link function: logarithmic, giving multiplicative means.

Probability distribution: Poisson

- b. What other probability distribution could you use for claim payments if you are not obliged to reproduce the predictions of the chain ladder method? Discuss why there could be better distributions for claim payments. [20 marks]

The gamma distribution is better suited to claim payments. Its standard deviation is proportional to the mean, while in the Poisson distribution the variance is proportional to the mean.

Now assume that instead of one portfolio, you have two portfolios to analyse.

The portfolios are closely related: Their payment patterns are identical; the year-to-year variation of the ultimate claim rates is driven by the same risk factors; however, the absolute size of the ultimate claim rates differs by a fixed relativity that you do not know.

You could think of the two portfolios as similar insurances of similar objects, where the only difference between the portfolios is in the size of the objects and/or the insurance cover and/or the premium rate.

Let us denote the earned premiums of the two portfolios by $p_j^{(1)}$ and $p_j^{(2)}$, and their claim payments by $X_{je}^{(1)}$ and $X_{je}^{(2)}$.

- c. Specify a GLM that will allow you to estimate the payment pattern, the year-to-year variation of the ultimate claim rates, and the fixed relativity between the ultimate claim rates, using all available data.

[30 marks]

Covariate structure: $\mu_{je}^{(k)} = \alpha_j + \beta_e + \gamma_k$

or $E(X_{je}^{(k)}) = p_j \exp(\alpha_j + \beta_e + \gamma_k)$

for $k=1,2$ (portfolios), $j=2010-2014$ (accident years), $e=0,...,4$ (development years).

THE END

Name: _____

Answer sheet for Question 2a

| | | Development year e | | | | |
|---|--|----------------------|-------|-------|-------|-------|
| | Specify formula | 0 | 1 | 2 | 3 | 4 |
| Incremental development factors δ_e^* | $\delta_e^* = \frac{\sum_{j=1}^{2014-e} X_{j,\leq e}}{\sum_{j=1}^{2014-e} X_{j,\leq e-1}}$ | | 1,696 | 1,055 | 1,031 | 1,007 |
| Cumulative development factors Δ_e^* | $\Delta_0^* = 1, \Delta_e^* = \prod_{e'=0}^e \delta_{e'}^*$ | 1,000 | 1,696 | 1,789 | 1,845 | 1,858 |
| Cumulative payment proportions $\pi_{\leq e}^*$ | $\pi_{\leq e}^* = \frac{\Delta_e^*}{\Delta_4^*}$ | 0,538 | 0,913 | 0,963 | 0,993 | 1,000 |
| Incremental payment proportions π_e^* | $\pi_e^* = \pi_{\leq e}^* - \pi_{\leq e-1}^*$ | 0,538 | 0,375 | 0,050 | 0,030 | 0,007 |

Name: _____

Answer sheet for Question 3a

| Accident year j | Earned premium | Cumulative Payments to date | $\pi(\text{cum.})$ | Theta (CL) | Outstanding payments (prediction) | Ultimate payments (prediction) |
|-------------------|----------------|-----------------------------|-----------------------|---|--|--------------------------------|
| Specify formula | p_j | $X_{j, \leq 2014-j}$ | $\pi_{\leq 2014-j}^*$ | $\theta_j^* = \frac{X_{j, \leq 2014-j}}{p_j \pi_{\leq 2014-j}^*}$ | $\bar{X}_{j, > 2014-j} = p_j \theta_j^* (1 - \pi_{\leq 2014-j}^*)$ | |
| 2010 | 153 | 140 | 1,000 | 0,915 | 0,000 | 140,000 |
| 2011 | 167 | 159 | 0,993 | 0,959 | 1,144 | 160,144 |
| 2012 | 191 | 152 | 0,963 | 0,826 | 5,861 | 157,861 |
| 2013 | 206 | 140 | 0,913 | 0,745 | 13,399 | 153,399 |
| 2014 | 219 | 93 | 0,538 | 0,789 | 79,828 | 172,828 |
| Total | 936 | 684 | | | 100,232 | 784,232 |

Name: _____

Answer sheet for Question 3b and 3c

| | Payment year | | | | |
|--|--------------|--------|-------|-------|---------|
| Accident year | 2015 | 2016 | 2017 | 2018 | Total |
| 2010 | 0,000 | 0,000 | 0,000 | 0,000 | 0,000 |
| 2011 | 1,144 | 0,000 | 0,000 | 0,000 | 1,144 |
| 2012 | 4,734 | 1,128 | 0,000 | 0,000 | 5,861 |
| 2013 | 7,703 | 4,600 | 1,096 | 0,000 | 13,399 |
| 2014 | 64,733 | 8,679 | 5,182 | 1,234 | 79,828 |
| Total payments during payment year (prediction) | 78,313 | 14,406 | 6,278 | 1,234 | 100,232 |
| Outstanding payments at start of payment year (prediction) | 100,232 | 21,919 | 7,513 | 1,234 | |
| Outstanding payments at end of payment year (prediction) | 21,919 | 7,513 | 1,234 | 0,000 | |

Name: _____

Answer sheet for Question 4a and 4c

| | Payment year | | | | Total |
|---|----------------|---------------|--------------|--------------|---|
| | 2015 | 2016 | 2017 | 2018 | |
| Predicted payments at 31 st December of the payment year | 78,313 | 14,406 | 6,278 | 1,234 | 100,232 |
| Discount factor at 31st December 2014 | 0,952 | 0,907 | 0,864 | 0,823 | |
| Discounted value of predicted payments at 31st December 2014 | 74,584 | 13,067 | 5,423 | 1,016 | "Best estimate" 94,090 |
| Outstanding payments on 1st January of the payment year | 100,232 | 21,919 | 7,513 | 1,234 | |
| Solvency capital required during the year | 50,116 | 10,959 | 3,756 | 0,617 | |
| Upfront cost of solvency capital provided during the year | 3,007 | 0,658 | 0,225 | 0,037 | 3,927 |
| Discount factor at 31st December 2014 | 1,000 | 0,952 | 0,907 | 0,864 | |
| Discounted value of cost of capital at 31st December 2014 | 3,007 | 0,626 | 0,204 | 0,032 | "Risk margin" 3,870 |