Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

DESDE 1911

Master in Actuarial Science

Loss Reserving

28-06-2013

Time allowed: 2 hours

Instructions:

1. This paper contains **5** questions and comprises **4** pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.

The following table shows cumulative paid claims for accident years 2008-2012 at 31.12.2012.

Cumulative claim payments

	Development year				
Accident year	0	1	2	3	4
2008	112	168	178	180	182
2009	116	180	190	192	
2010	140	228	246		
2011	156	262			
2012	170				

The exposure is shown in the next table.

Accident year	Exposure
2008	244
2009	252
2010	296
2011	306
2012	334

1. <u>Preparation</u>

   a. Organise the incremental paid claims in a development triangle. [5 marks]

Solution

Incremental claim statistics

	Development year				
Accident year	**0**	**1**	**2**	**3**	**4**
2007	**112**	**56**	**10**	**2**	**2**
2008	**116**	**64**	**10**	**2**	
2009	**140**	**88**	**18**		
2010	**156**	**106**			
2011	**170**				

2. <u>Bornhuetter-Ferguson method</u>

   In what follows, claim ratio means claims divided by exposure. You may assume that no claims will be paid with more than 4 years' delay.

   a. Estimate the delay-specific claim ratios for delays 0 to 4. [10 marks]

Solution

Empirical claim ratios

	Development year				
Accident year	**0**	**1**	**2**	**3**	**4**
2007	**45,9 %**	**23,0 %**	**4,1 %**	**0,8 %**	**0,8 %**
2008	**46,0 %**	**25,4 %**	**4,0 %**	**0,8 %**	
2009	**47,3 %**	**29,7 %**	**6,1 %**		
2010	**51,0 %**	**34,6 %**			
2011	**50,9 %**				
Average	**48,5 %**	**28,6 %**	**4,8 %**	**0,8 %**	**0,8 %**

b. Estimate the overall claim ratio per accident year (all delays). [10 marks]

Solution

83.5% = 48.5 % + 28.6 % + 4.8 % + 0.8 % + 0.8 %

c. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]

Solution

Payment pattern	**0**	**1**	**2**	**3**	**4**
Cumulative	**58,05 %**	**92,31 %**	**98,05 %**	**99,02 %**	**100,00 %**
Incremental	**58,05 %**	**34,25 %**	**5,75 %**	**0,97 %**	**0,98 %**

Incremental = Claim ratio per delay / 83.5%

d. Estimate the outstanding claim payments for each accident year. [10 marks]

e. Calculate the estimated ultimate claim cost and the estimated ultimate claim ratio of each accident year. [10 marks]

Solution d-e

Accident year	Exposure	Paid claims	pi(cum.)	Theta_BF	**Outstanding**	**Ultimate**	**Loss ratio**
2007	244	182	100,0 %	83,5 %	**0,0**	**182**	**74,6 %**
2008	252	192	99,0 %	83,5 %	**2,1**	**194**	**77,0 %**
2009	296	246	98,1 %	83,5 %	**4,8**	**251**	**84,7 %**
2010	306	262	92,3 %	83,5 %	**19,7**	**282**	**92,0 %**
2011	334	170	58,1 %	83,5 %	**117,0**	**287**	**85,9 %**
Total	1 432	1 052			**143,5**	**1 196**	**83,5 %**

3. Bühlmann-Straub model

a. Describe briefly the assumptions of the Bühlmann-Straub model for claim amounts and explain the meaning of the parameters β, φ and λ. [15 marks]

Solution

The candidate should mention:

- **Conditional on an unobserved risk parameter that we denote by Θ_j, the incremental payments $X_{j0}, X_{j1}, \cdots$ are stochastically independent with conditional mean $\mathrm{E}(X_{je} \mid \Theta_j) = p_j b(\Theta_j)\pi_e$ and variance $\mathrm{Var}(X_{je} \mid \Theta_j) = p_j v(\Theta_j)\pi_e$.**
- **The unobserved risk parameter Θ_j is seen as the outcome of a random variable.**
- **The $\Theta_1, \cdots, \Theta_J$ are stochastically independent and identically distributed. We denote the mean and variance of the function $b(\Theta_j)$ by $\beta = \mathrm{E}(b(\Theta_j))$ and $\lambda = \mathrm{Var}(b(\Theta_j))$. We denote the mean of the function $v(\Theta_j)$ by $\varphi = \mathrm{E}(v(\Theta_j))$.**

The optimal credibility estimator of the random claim level of accident year *j* has the form $\bar{b}_j = \zeta_j \hat{b}_j + (1 - \zeta_j)\beta$, where $\hat{b}_j$ is the chain ladder estimator, β is the prior mean, and ζ_j is the optimal credibility factor.

b. Specify the formula for the optimal credibility factor ζ_j. [10 marks]

Solution

Optimal credibility factor $\zeta_j = \dfrac{\lambda p_j \pi_{\leq J-j}}{\lambda p_j \pi_{\leq J-j} + \varphi}$

c. Explain in what way the Bornhuetter-Ferguson method and the chain ladder method can be seen as limiting cases of the Bühlmann-Straub credibility method. [10 marks]

Solution

Bornhuetter-Ferguson is limiting case for $\lambda \to 0$ **or** $\varphi \to \infty$ **.**

Chain ladder is a limiting case for $\lambda \to \infty$ **or** $\varphi \to 0$

Now assume that the following parameter values have been estimated:

β (beta)	0.83
φ (phi)	0,3
λ (lambda)	0.004

d. Use these parameter values and the payment pattern from problem 2 to complete the following table. Please specify the formula that you use in each column.

Accident year	Exposure	Claims paid	$\pi_{\le J-j}$	$\hat{b}_j$	β	ζ_j	$\bar{b}_j$	Outstanding	Ultimate	Claims ratio
2008	244	182								
2009	252	192								
2010	296	246								
2011	306	262								
2012	334	170								
Total	1 432	1 052								

[20 marks]

Solution

Accident year	Exposure	Claims paid	$\pi_{\le J-j}$	$\hat{b}_j$	β	ζ_j	$\bar{b}_j$	Outstanding	Ultimate	Loss ratio
2007	244	182	100,0 %	74,6 %	83,0 %	76,5 %	76,6 %	0	182	75 %
2008	252	192	99,0 %	76,9 %	83,0 %	76,9 %	78,3 %	2	194	77 %
2009	296	246	98,1 %	84,8 %	83,0 %	79,5 %	84,4 %	5	251	85 %
2010	306	262	92,3 %	92,8 %	83,0 %	79,0 %	90,7 %	21	283	93 %
2011	334	170	58,1 %	87,7 %	83,0 %	72,1 %	86,4 %	121	291	87 %
Total	1 432	1 052		83 %				149	1 201	84 %

Formulas

$\pi_{\le J-j}$ **= cumulative payment proportion**

$\hat{b}_j$ **= Paid claims / (Exposure x** $\pi_{\le J-j}$ **)**

β **= given**

$$\zeta_j = \frac{\lambda p_j \pi_{\le J-j}}{\lambda p_j \pi_{\le J-j} + \varphi}$$

$$\bar{b}_j = \zeta_j \hat{b}_j + (1-\zeta_j)\beta$$

Outstanding = $p_j \bar{b}_j (1-\pi_{\le J-j})$

Ultimate **= Paid claims + Outstanding**

Claims ratio **= Ultimate / Exposure**

4. Generalised linear models

A generalized linear model (GLM) is fully specified by the following three choices: The link function, the covariate structure and the probability distribution.

a. Specify the link function that will provide multiplicative means. [10 marks]

Solution

Logarithmic link function, ln E(X) = m'b, m being covariates and b a vector of regression coefficients

b. Specify a covariate structure that will allow you to estimate a multiplicative model that has development year effects and an inflation rate acting on calendar years. It is sufficient to write down the formula, no matrix is required. [20 marks]

Solution

$$\mu_{je} = \alpha + \beta_e + \lambda \cdot (j + e - 1)$$

J = accident year, e = development year

5. Stages in the life of a claim

a. Explain the meaning of the acronyms RNBS, IBNR and CBNI. Please do not just translate the abbreviations, but explain what it means for a claim to be "RBNS", "IBNR" or "CBNI" on a specific valuation date. [10 marks]

Solution

RBNS, reported but not settled. Used in the meaning of "reported", as settled claims may be reopened.

IBNR, incurred but not reported. The loss leading to the claims has occurred on the valuation date, but we haven't received the claim yet.

CBNI, covered but not incurred. The loss leading to the claims will occur (and be reported) after the valuation date, and the insurer will be liable under a policy already issued.

b. Suggest a few pieces of information that could be useful in modeling the development and estimating the ultimate cost of claims that are RBNS. [10 marks]

Solution

The number of claims, the nature of those claims, the handling and settlement stage of each claim, the age, income and health condition of the claimant, etc. etc.

c. Suggest what information could be used to model the arrival and estimate the ultimate cost of claims that are (still) IBNR. [10 marks]

Solution

Essentially, just the risk exposure, claim frequency and severity distribution.

d. Explain the meaning of this assertion:
"Statistically, CBNI claims behave in the same way as IBNR claims." [15 marks]

Solution

IBNR and CBNI claims are both unreported, thus unknown. All we know is that they arrive with a certain statistical regularity. To model their cost we must estimate risk exposure, claim frequency and severity distribution.

e. Explain the meaning of this assertion:
"Know your RBNS, then IBNR/CBNI come by themselves (well, almost)." [15 marks]

Solution

The only way to model the cost of unreported claims (IBNR/CBNI) is to analyse carefully the cost of reported claims (RBNS). Having done the latter, estimating the former reduces to an exposure x frequency x severity calculation.