Master in Actuarial Science

Loss Reserving

07-07-2017

Time allowed: 2 hours

Instructions:

1. This paper contains **13** questions and comprises **3** pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.

We have observed claims (counts or payments) for accident years $j = 1, \cdots, J$, where J denotes the current year. The set of observed, incremental claims is $\{X_{je} : j = 1, \cdots, J, e = 0, \cdots, J - j\}$, where X_{je} denotes claims from accident year j that emerge in calendar year $j + e$. The amount of risk exposed in accident year *j* has been p_j. The expected ultimate claim rate relative to the amount of risk we denote by θ_j and the proportion of claims emerging after *e* years, we denote by π_e.

Let us for argument's sake assume that the X_{je} are stochastically independent and follow a Poisson distribution with expected value $\mathrm{E}(X_{je}) = p_j \cdot \theta_j \cdot \pi_e$.

1. Prove that the maximum likelihood estimators of θ_j and π_e must satisfy the equations

   $X_{j,\le J-j} = p_j \cdot \theta_j^* \cdot \pi_{\le J-j}^*$ for $j = 1, \cdots, J$, and

   $X_{\le J-e,e} = \left(\sum_{j=1}^{J-e} p_j \cdot \theta_j^* \right) \cdot \pi_e^*$ for $e = 0, \cdots, J-1$.

   [25 marks]

The equations in (1) cannot be solved directly and explicitly, but require a recursion. There is a simpler way. Define development factors $\delta_e^* = \pi_{\le e}^* / \pi_{\le e-1}^*$ and $\delta_0^* = 1$.

2. Prove that $\pi_{\le e}^* = \prod_{e'=0}^{e} \delta_{e'}^* / \prod_{e'=0}^{J-1} \delta_{e'}^*$ [10 marks]
3. Write down how you can calculate the maximum likelihood estimates θ_j^* and π_e^*, starting with the empirical development factors $\delta_e^* = \sum_{j \le e} X_{j,\le e} / \sum_{j \le e} X_{j,\le e-1}$.

   [20 marks]

4. Calculate the estimates $\{\theta_j^* : j = 1, \cdots, 4\}$ and $\{\pi_d^* : d = 0, \cdots, 3\}$ from the following data:

Accident year	Exposure
1	5124
2	4719
3	3898
4	3575

Claim counts	Development year			
Accident year	0	1	2	3
1	87	58	8	3
2	77	45	8	
3	79	41		
4	84			

   [25 marks]

5. Use the chain ladder method to predict the future claims. [10 marks]

6. Use the Cape Cod method to calculate a weighted average of $\{\theta_j^* : j = 1, \cdots, 4\}$ and denote it by θ^*. [10 marks]

Benktander's method is characterised by the prediction $\overline{X}_{j,>J-j} = p_j \overline{\theta}_j \pi_{>J-j}$,
where $\overline{\theta}_j = \pi_{\leq J-j}\theta_j^* + (1-\pi_{\leq J-j})\theta^*$ is a weighted average.

7. Complete the following table, using Benktander's method.

j	p_j	$N_{j,\leq J-j}$	$\pi_{\leq J-j}$	θ_j^*	$\overline{\theta}_j$	$\overline{X}_{j,>J-j}$
1	5 124	156	100 %			
2	4 719	130	98 %			
3	3 898	120	93 %			
4	3 575	84	58 %			
Total	17 316	490				

[20 marks]

8. Explain the advantages of Benktander's method, compared with the chain ladder method. [10 marks]

9. Describe some methods that you can use to smooth or extrapolate the development pattern in the tail, where data is scarce or unavailable. [10 marks]

10. Explain the meaning of the acronyms CBNI, IBNR and RBNS, then explain why "IBNR has more in common with CBNI than with RBNS". [20 marks]

Generalised linear models (GLM) can be used to model many different structures.

11. Describe the three components that define a GLM. [10 marks]

12. Using GLM, propose a (simple!) joint model of two insurance portfolios. The portfolios are motor insurance in two different countries or regions. You assume that they have the same development pattern but different claim rates. You want to use the total statistical information from the two portfolios to estimate the development pattern and the two claim rates. [20 marks]

13. Explain the meaning of the assertion "Every claim cohort is a different portfolio when you use the chain ladder method". [10 marks]

END