Master in Actuarial Science

Loss Reserving

21-06-2016

Time allowed: 2 hours

Instructions:

1. This paper contains **5** questions and comprises **3** pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.
11. Preprinted answer sheets are available for some of the tables required.

The following data shows paid claims for the period 2007-2011 at 31.12.2011.

Incremental	Payment delay in years				
Accident year	0	1	2	3	4
2007	56	28	5	1	1
2008	58	32	5	1	
2009	70	44	9		
2010	78	53			
2011	85				

Cumulative	Payment delay in years				
Accident year	0	1	2	3	4
2007	56	84	89	90	91
2008	58	90	95	96	
2009	70	114	123		
2010	78	131			
2011	85				

The premium is shown in the next table.

Accident year	Premium
2007	122
2008	126
2009	148
2010	153
2011	167

You may assume that no claims will be paid with a delay of more than four years.

1. Bornhuetter-Ferguson method
   a. Estimate the delay-specific claim ratios. [10 marks]
   b. Estimate the average claim ratio per accident year (all delays). [10 marks]
   c. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]
   d. Estimate the outstanding claim payments for each accident year. [10 marks]
   e. Fill the missing cells in the incremental run-off triangle with predictions. [10 marks]

a. Calculation: sum of incremental claims per column / sum of corresponding exposures.

	0	1	2	3	4
Claim ratio	**48,5%**	**28,6%**	**4,8%**	**0,8%**	**0,8%**

b. Calculation: sum of claim ratios for delays 0-4 in a.

Average	**83,5 %**

c. Calculation: Results from a) divided by result in b).

	0	1	2	3	4
Cumulative pi(cum.)	58,05 %	92,31 %	98,05 %	99,02 %	100,00 %
Incremental pi(inc.)	58,05 %	34,25 %	5,75 %	0,97 %	0,98 %

d. Calculation: Outstanding = Exposure x Loss ratio x (1-pi(cum.))

Accident year	Exposure	Developed to	Observed	pi(cum.)	Loss ratio	Outstanding
2007	122	4	91	100,0 %	83,5 %	0
2008	126	3	96	99,0 %	83,5 %	1
2009	148	2	123	98,1 %	83,5 %	2
2010	153	1	131	92,3 %	83,5 %	10
2011	167	0	85	58,1 %	83,5 %	58
Total	716		526			72

e. Calculation: Outstanding = Exposure x Loss ratio x pi(inc.)

	Development year				
Accident year	0	1	2	3	4
2 007					
2 008					1
2 009				1	1
2 010			7	1	1
2 011		48	8	1	1

2. Chain ladder method
   a. Estimate the year-on-year development factors. [10 marks]
   b. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]
   c. Estimate the overall claim ratio for each accident year. [10 marks]
   d. Estimate the outstanding claim payments for each accident year. [10 marks]
   e. Fill the missing cells in the incremental run-off triangle with predictions. [10 marks]

a. **Calculation: Sum of cumulative claims per column / sum of corresponding cumulative claims in previous column.**

Empirical	0	1	2	3	4
Average		159,92 %	106,60 %	101,09 %	101,11 %

b. **Calculation: Accumulate development factors to end, divide accumulated factors for each delay by ultimate**

delta to pi	0	1	2	3	4
Devt. factor (incr.)		1,5992	1,0660	1,0109	1,0111
Devt. factor (cum.)	100 %	160 %	170 %	172 %	174 %
pi (cum.)	57,39 %	91,78 %	97,84 %	98,90 %	100,00 %
pi (incr.)	57,39 %	34,39 %	6,06 %	1,06 %	1,10 %

c. **Calculation for c. Observed / (Exposure x pi(cum.))**

d. **Calculation for d. Calculation: Outstanding = Exposure x Loss ratio x (1-pi(cum.))**

Accident year	Exposure	Developed to	Observed	pi(cum.)	c. Loss ratio	d. Outstanding
2007	122	4	91	100 %	74,6 %	0
2008	126	3	96	99 %	77,0 %	1
2009	148	2	123	98 %	84,9 %	3
2010	153	1	131	92 %	93,3 %	12
2011	167	0	85	57 %	88,7 %	63
Total	716		526		83,8 %	79

e. **Calculation: Outstanding = Exposure x Loss ratio x pi(inc.)**

	Development year				
Accident year	0	1	2	3	4
2007					
2008					1
2009				1	1
2010			9	2	2
2011		51	9	2	2

3. Other questions
   a. Would you characterize the payments as short-tailed or long-tailed? [10 marks]
   b. You have used premium as a measure of risk exposure, but premium is not always a reliable measure. Suggest one other measure of risk exposure that you can easily use in this situation. [10 marks]
   c. Describe in words a method that you can use to combine the estimates from Bornhuetter-Ferguson and chain ladder. No formulas are needed. [10 marks]

Answers:

a. Short-tailed

b. One could use payments in the accident year (at delay 0) to measure exposure

c. Keywords: credibility, Benktander, Bühlmann-Straub

4. Reproducing known methods by GLM

A GLM is fully specified by its covariate structure, its link function and its probability distribution.

a. Specify a GLM that will reproduce the predictions of the chain ladder method. [10 marks]

Covariate structure: $\mu_{je} = \alpha_j + \beta_e$ **or equivalently, accident year and development year as classification variables.**

Link function: logarithmic, giving multiplicative means.

Probability distribution: Poisson

b. Specify a GLM that will reproduce the predictions of the Bornhuetter-Ferguson method. [10 marks]

Covariate structure: $\mu_{je} = \beta_e$ **or equivalently, development year as classification variable.**

Link function: logarithmic, giving multiplicative means.

Probability distribution: Poisson or Gamma

c. What probability distribution would you use for claim payments If your intention is not to reproduce a certain method? [10 marks]

The gamma distribution is better suited to claim payments. Its standard deviation is proportional to the mean, while in the Poisson distribution the variance is proportional to the mean.

5. Modelling by GLM

You have two portfolios that are closely related:

- Their payment patterns are similar;
- The claim rates are driven by the same accident year inflation;
- The absolute size of the ultimate claim rates differs by a fixed relativity.

You could think of the two portfolios as similar insurances of similar objects, where the only difference between the portfolios is in the size of the objects and/or the insurance cover and/or the premium rate.

Let use denote the earned premiums of the two portfolios by $p_j^{(1)}$ and $p_j^{(2)}$, and their claim payments by $X_{je}^{(1)}$ and $X_{je}^{(2)}$.

a. Specify a GLM that will allow you to estimate the payment pattern, the inflation of the claim rates, and the fixed relativity between the ultimate claim rates, using all available data. Explain all the symbols and parameters that you use. [40 marks]

Covariate structure: $\mu_{je}^{(k)} = \alpha(j-1) + \beta_e + \gamma_k$

or $E\left(X_{je}^{(k)}\right) = p_j \exp\left(\alpha(j-1) + \beta_e + \gamma_k\right)$

for *k*=1,2 (portfolios), *j*=2007-2011 (accident years), *e*=0,…,4 (development years).