



Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

DESDE 1911

Master in Actuarial Science

Loss Reserving

07-06-2013

Time allowed: 2 hours

Instructions:

1. This paper contains **6** questions and comprises **4** pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.

You are the actuary of a general insurance company and have been asked to analyse the claim cost of a certain type of insurance. The company has been active in that line of insurance for five years only, and has systematically collected payment data only during the last three years.

This is the data you receive from the company:

Cumulative paid claims

Accident year	Valuation date		
	31.12.2010	31.12.2011	31.12.2012
2008	189	266	324
2009	113	235	293
2010	23	99	152
2011		78	186
2012			54

Accident year	Earned premium
2008	437
2009	463
2010	503
2011	587
2012	659

1. Preparation

- a. Organise the paid claims in an (incomplete) development triangle. [10 marks]

Solution

Accident year	Development year				
	0	1	2	3	4
2008			189	266	324
2009		113	235	293	
2010	23	99	152		
2011	78	186			
2012	54				

2. Chain ladder method

- a. Estimate the year-on-year development factors for development years 1 to 4. [10 marks]

Solution

Accident year	Development year				
	0	1	2	3	4
2008				140,74 %	121,80 %
2009			207,96 %	124,68 %	
2010		430,43 %	153,54 %		
2011		238,46 %			
2012					
Average		282,18 %	182,55 %	131,84 %	121,80 %

- b. Assume that claims paid up to development year 4 make up 90% of ultimate claim cost. As a convenience, you may assume that the remaining 10% is paid in development year 5.

Calculate the development factor for development year 5.

[10 marks]

Solution

$$100/90 = 111.11\%$$

- c. Estimate the payment pattern expressed in percent of ultimate cost.

[10 marks]

Solution

Convert delta to pi	Development year					
	0	1	2	3	4	5
Dev. factor (incr.)		2,8218	1,8255	1,3184	1,2180	1,1111
Dev. factor (cum.)	100 %	282 %	515 %	679 %	827 %	919 %
Payment pattern (cum.)	10,88 %	30,70 %	56,04 %	73,89 %	90,00 %	100,00 %
Payment pattern (incr.)	10,88 %	19,82 %	25,34 %	17,84 %	16,11 %	10,00 %

- d. Estimate the outstanding claim payments for each accident year.

[10 marks]

- e. Calculate the estimated ultimate claim cost and the estimated ultimate loss ratio of each accident year.

[10 marks]

Solution (2d-e)

Accident year	Premium	Developed to	Paid claims	pi(cum.)	Theta (CL)	Outstanding	Ultimate	Loss ratio
2008	437	4	324	90 %	82,4 %	36	360	82 %
2009	463	3	293	74 %	85,6 %	104	397	86 %
2010	503	2	152	56 %	53,9 %	119	271	54 %
2011	587	1	186	31 %	103,2 %	420	606	103 %
2012	659	0	54	11 %	75,3 %	442	496	75 %
Total	2 649		1 009		79,5 %	1 121	2 130	80 %

3. Benktander method

You may use earned premium as a measure of risk exposure.

- a. Calculate the average loss ratio using the “Cape Cod” method.

[10 marks]

Solution

Theta (BF) = $\text{sum}(\text{paid}) / \text{sumproduct}(\text{exposure}, \text{development proportion})$
= $1009 / (437 \times 90\% + 463 \times 74\% + 503 \times 56\% + 587 \times 31\% + 659 \times 11\%) = 79.5\%$
(see table for 2 d-e)

- b. Estimate the outstanding claim payments using Benktander’s method. Please organize your calculations in a table with the following contents. The columns “Theta” signify the estimated loss ratio for the tail, as estimated by chain ladder (CL), Cape Cod method (BF), and Benktander’s weighted average. For each column, please show the formula.

Accident year	Earned premium	Paid claims	Paid proportion	Theta (CL)	Theta (BF)	Theta (Benkt.)	Outstanding payments (Benktander)	Ultimate claim cost (Benktander)	Loss ratio (Benktander)
2008									
2009									
2010									
2011									
2012									
Total									

[20 marks]

Solution

Accident year	Premium	Paid claims	Paid proportion	Theta(CL)	Theta (BF)	Theta (Benkt.)	Outstanding	Ultimate	Loss ratio
2008	437	324	90 %	82,4 %	79,5 %	82,1 %	36	360	82 %
2009	463	293	74 %	85,6 %	79,5 %	84,0 %	102	395	85 %
2010	503	152	56 %	53,9 %	79,5 %	65,2 %	144	296	59 %
2011	587	186	31 %	103,2 %	79,5 %	86,8 %	353	539	92 %
2012	659	54	11 %	75,3 %	79,5 %	79,0 %	464	518	79 %
Total	2 649	1 009		79,5 %			1 099	2 108	80 %

Theta (CL) = Paid claims / (Premium x Paid proportion)

Theta (BF), see 3a

Theta (Benkt.) = Paid proportion x Theta (CL) + (1 – Paid proportion) x Theta (BF)

Outstanding = Premium x Theta (Benkt.) x (1 – Paid proportion)

Ultimate = Paid claims + Outstanding

Loss ratio = Ultimate / Premium

4. Projections and discounting

The valuation date is 31.12.12 and the discount rate is 5%.

- a. Use the estimated outstanding payments of the Benktander method and the estimated payment pattern to complete the table of projected incremental claim payments. Please show the formula you used.

Incremental paid claims		Payment year							
Accident year		2010	2011	2012	2013	2014	2015	2016	2017
2008	#N/A	77	58	?					
2009	#N/A	122	58	?	?				
2010	23	76	53	?	?	?			
2011		78	108	?	?	?	?		
2012		54		?	?	?	?	?	?
Total	#N/A	353	331	?	?	?	?	?	?

[10 marks]

Solution

Accident year	2 010	2 011	2 012	2 013	2 014	2 015	2 016	2 017
2 008	#N/A	77	58	36	0	0	0	0
2 009	#N/A	122	58	63	39	0	0	0
2 010	23	76	53	58	53	33	0	0
2 011		78	108	129	91	82	51	0
2 012		54		103	132	93	84	52
Sum	#N/A	353	331	389	315	208	135	52

Projection = Premium x Theta (Benkt.) x Incremental paid proportion

- b. Calculate the discounted value of outstanding claim payments. You may assume that all payments are made at the end of the payment year. [10 marks]

Solution

Discount rate	5,0 %					
	2 013	2 014	2 015	2 016	2 017	Total
Discount factor	95,2 %	90,7 %	86,4 %	82,3 %	78,4 %	
Payments	389	315	208	135	52	1 099
Discounted value	371	285	179	111	41	987

The valuation date is still 31.12.12. Assume that earned premium for 2013 will be 750 and that the expected loss ratio for 2013 is 70%.

- c. Calculate the expected nominal value of claims incurring in accident year 2013. [5 marks]

Solution

$$750 \times 70\% = 525$$

- d. Calculate the expected discounted value of claims incurring in accident year 2013. [10 marks]

Solution

Discount rate	5,0 %						
	2 013	2 014	2 015	2 016	2 017	2 018	Total
Discount factor	95,2 %	90,7 %	86,4 %	82,3 %	78,4 %	74,6 %	
Nominal value	57	104	133	94	85	53	525
Discounted value	54	94	115	77	66	39	446

5. Other information

- a. Discuss what other information it could be reasonable to require from the company in order to assess the risk exposure. [10 marks]

Solution

Number of policies, number of insured, number of claims

- b. Discuss what other information it could be reasonable to require from the company in order to assess the tail of payments beyond development year 4. [10 marks]

Solution

Outstanding case estimates, number of open claims, attachment conditions

- c. Explain what is meant by the policy attachment conditions “claims incurred” and “claims made”. [15 marks]

Solution

Claims incurred: policy covers a claim if the loss leading to the claim occurred during the policy period.

Claims made: policy covers claims made (reported) during the policy period.

- d. With the data you have received (premiums and claim payments only), will information about the attachment condition of the policies be useful? Please answer yes or no and explain your answer briefly. [10 marks]

Solution

Not useful. If only payments are provided as data there is no way of telling whether development of payments is due to reported claims being handled/settled or new claims being reported.

6. Generalised linear models

A generalized linear model (GLM) is fully specified by the following three choices: The link function, the covariate structure and the probability distribution.

- a. Specify the link function that will provide multiplicative means. [10 marks]

Solution

Logarithmic link function, $\ln E(X) = m'b$, m being covariates and b a vector of regression coefficients

- b. Specify a covariate structure that will allow you to estimate a multiplicative model with accident year effects and development year effects. It is sufficient to write down the formula, no matrix is required. [10 marks]

Solution: $\ln E(X_{jd}) = \alpha_j + \beta_d$ (j =accident year, d =development year)

- c. Give an example of a probability distribution from the exponential family that could reasonably be used to model claim payments. [5 marks]

Solution: Gamma

- d. What probability distribution would you use if your aim was to model claim numbers? [5 marks]

Solution: Poisson