Master in Actuarial Science
Loss Reserving
03-06-2011
Time allowed: 2 hours

Model solution

Instructions:

1. This paper contains 7 questions and comprises 3 pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all 7 questions.
6. Begin your answer to each of the 7 questions on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.

You are the actuary of a general insurance company and have received the following data showing paid claims on 31.12.2008.

Incremental	Payment delay				
Accident year	0	1	2	3	4
2004	0	13	75	555	1142
2005	4	23	894	4734	
2006	3	14	195		
2007	1	11			
2008	0				

Cumulative	Payment delay				
Accident year	0	1	2	3	4
2004	0	13	88	643	1785
2005	4	27	921	5655	
2006	3	17	212		
2007	1	12			
2008	0				

The exposure is shown in the next table.

Accident year	Exposure
2004	17050
2005	17250
2006	17200
2007	17500
2008	17200

You may assume that no claims will be paid with a delay of more than four years.

1. Bornhuetter-Ferguson method
   a. Estimate the delay-specific claim rates. By claim rate we mean claim payments per unit of exposure. [10 marks]
   b. Estimate the overall claim rate per accident year. [10 marks]
   c. Estimate the payment pattern. [10 marks]
   d. Estimate the outstanding claim payments for each accident year. [10 marks]
   e. Fill the missing cells in the run-off triangle with predictions. [10 marks]

a.

e	0	1	2	3	4
θ_e^*	0,000093	0,000884	0,022602	0,154198	0,066979

b.

θ^*	0,244757

c.

e	0	1	2	3	4
$\pi^*_{\le e}$ (cumulative)	0,04 %	0,40 %	9,63 %	72,63 %	100,00 %
π^*_e (incremental)	0,04 %	0,36 %	9,23 %	63,00 %	27,37 %

d.

Accident year	Exposure	Developed to	Observed	pi(cum.)	θ^*	Outstanding	Ultimate
2004	17050	4	1785	100,00 %	2,45E-01	0	1 785
2005	17250	3	5655	72,63 %	2,45E-01	1 155	6 810
2006	17200	2	212	9,63 %	2,45E-01	3 804	4 016
2007	17500	1	12	0,40 %	2,45E-01	4 266	4 278
2008	17200	0	0	0,04 %	2,45E-01	4 208	4 208
Total	86200		7664			13 434	21 098

e.

Accident year	0	1	2	3	4
2004	0	13	75	555	1142
2005	4	23	894	4734	1155
2006	3	14	195	2652	1152
2007	1	11	396	2698	1172
2008	0	15	389	2652	1152

2. Chain ladder method
   a. Estimate the development factors. [10 marks]
   b. Estimate the payment pattern. [10 marks]
   c. Estimate the overall claim rate per accident year. [10 marks]
   d. Estimate the outstanding claim payments for each accident year. [10 marks]
   e. Fill the missing cells in the run-off triangle with predictions. [10 marks]

a.

e	0	1	2	3	4
δ^*_e		862,50 %	2142,11 %	624,18 %	277,60 %

b.

e	0	1	2	3	4
$\pi^*_{\le e}$ (cumulative)	0,03 %	0,27 %	5,77 %	36,02 %	100,00 %
π^*_e (incremental)	0,03 %	0,24 %	5,50 %	30,25 %	63,98 %

c. and

d.

Accident year	Exposure	Developed to	Observed	pi(cum.)	Overall θ^*_j	Outstanding	Ultimate
2004	17050	4	1785	100,00 %	0,104692	0,0	1 785,0
2005	17250	3	5655	36,02 %	0,910062	10 043,6	15 698,6
2006	17200	2	212	5,77 %	0,213573	3 461,5	3 673,5
2007	17500	1	12	0,27 %	0,254520	4 442,1	4 454,1
2008	17200	0	0	0,03 %	0,000000	0,0	0,0
Total	86200		7664		0,315274	17 947,1	25 611,1

e.

Accident year	0	1	2	3	4
2004	0	13	75	555	1142
2005	4	23	894	4734	10044
2006	3	14	195	1111	2350
2007	1	11	245	1347	2850
2008	0	0	0	0	0

3. Benktander's method

With claim rates and payment pattern from question 1, apply Benktander's method to estimate the outstanding claim payments for each accident year. [20 marks]

Accident year	Exposure	Developed to	Observed	pi(cum.)	Theta_BF	Theta_CL	Credibility z	Theta_bar	Outstanding
2004	17050	4	1785	100,00 %	2,45E-01	1,05E-01	100,00 %	1,05E-01	0
2005	17250	3	5655	72,63 %	2,45E-01	4,51E-01	72,63 %	3,95E-01	1 864
2006	17200	2	212	9,63 %	2,45E-01	1,28E-01	9,63 %	2,34E-01	3 629
2007	17500	1	12	0,40 %	2,45E-01	1,72E-01	0,40 %	2,44E-01	4 261
2008	17200	0	0	0,04 %	2,45E-01	0,00E+00	0,04 %	2,45E-01	4 207
Total	86200		7664						13 961

4. Choice of method
   a. Explain the properties of the Bornhuetter-Ferguson method and the chain ladder method (robustness, sensitivity). [10 marks]
   b. Which method would you choose for the portfolio shown here, and why? [10 marks]

   a. Bornhuetter-Ferguson: robust, not sensitive to change
      Chain ladder: volatile, sensitive to change
   b. Definitely not chain ladder, too sensitive.

5. Discounting

For the Bornhuetter-Ferguson method and using the predictions in 1.e:

a. Calculate the total predicted payments per future payment year. [10 marks]

b. Calculate the discounted value of future payments using 3% interest. [10 marks]

You may assume that all payments are made at the end of each year.

a.	2009	2010	2011	2012	Total
2004	0	0	0	0	0
2005	1155	0	0	0	1155
2006	2652	1152	0	0	3804
2007	396	2698	1172	0	4266
2008	15	389	2652	1152	4208
	4218	4239	3824	1152	13434
b.					
0,03					
1	97,09 %	94,26 %	91,51 %	88,85 %	
Discounted	4 095	3 996	3 500	1 024	12615

6. Other models
   a. Specify the mathematical assumptions of the Bühlmann-Straub model [20 marks]
   b. Specify the mathematical assumptions of the Mack model [10 marks]

a. The assumptions of Bühlmann-Straub's model are:

- Conditional on a risk parameter that we denote by Θ_j, the increments (payments) $X_{j0}, X_{j1}, \cdots$ are stochastically independent with conditional mean $\mathrm{E}(X_{je} \mid \Theta_j) = p_j b(\Theta_j)\pi_e$ and variance $\mathrm{Var}(X_{je} \mid \Theta_j) = p_j v(\Theta_j)\pi_e$.
- The quantity p_j denotes an observed measure of risk exposure, while the quantity π_e denotes the expected amount of increment in development year *e*.
- The unobserved risk parameter Θ_j is the outcome of a random variable.
- The risk parameters $\Theta_1, \cdots, \Theta_J$ are stochastically independent and identically distributed.
- The sets $\{\Theta_j, X_{j0}, X_{j1}, \cdots\}$ and $\{\Theta_k, X_{k0}, X_{k1}, \cdots\}$ are independent for different accident years (i.e., $j \neq k$).

b. The assumptions of Mack's model are ($\tilde{X}_{je}$ denotes accumulated payments):

1. There exist constants $\delta_1, \delta_2, \cdots, \delta_D$ such that $\mathrm{E}(\tilde{X}_{je} \mid \mathcal{D}_{j,e-1}) = \delta_e \tilde{X}_{j,e-1}$ for $e = 1, \cdots, D$.
2. There exist constants $\gamma_1, \gamma_2, \cdots, \gamma_D$ such that $\mathrm{Var}(\tilde{X}_{je} \mid \mathcal{D}_{j,e-1}) = \gamma_e \tilde{X}_{j,e-1}$ for $e = 1, \cdots, D$.
3. The ensembles $\mathcal{D}_{j,D}$ and $\mathcal{D}_{k,D}$ (accident years) are stochastically independent for $j \neq k$.

7. Claim categories

Explain the meaning of the abbreviations RBNS, IBNR and CBNI in such a way that a non-actuary understands them. You may use an illustration if you wish. [10 marks]

RBNS means Reported but not Settled, but in this course it has been used in the sense of Reported. IBNR means Incurred but not Reported, CBNI means Covered but not Incurred.