Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

DESDE 1911

Master in Actuarial Science

Loss Reserving

15-06-2012

Time allowed: 2 hours

Solution

Instructions:

1. This paper contains 6 questions and comprises 3 pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.

The following data shows paid claims for the period 2007-2011 at 31.12.2011.

Incremental	Payment delay in years				
Accident year	0	1	2	3	4
2007	56	28	5	1	1
2008	58	32	5	1	
2009	70	44	9		
2010	78	53			
2011	85				

Cumulative	Payment delay in years				
Accident year	0	1	2	3	4
2007	56	84	89	90	91
2008	58	90	95	96	
2009	70	114	123		
2010	78	131			
2011	85				

The premium is shown in the next table.

Accident year	Premium
2007	122
2008	126
2009	148
2010	153
2011	167

You may assume that no claims will be paid with a delay of more than four years.

1. Bornhuetter-Ferguson method
   a. Estimate the delay-specific claim ratios. [10 marks]
   b. Estimate the average claim ratio per accident year (all delays). [10 marks]
   c. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]
   d. Estimate the outstanding claim payments for each accident year. [10 marks]
   e. Fill the missing cells in the incremental run-off triangle with predictions. [10 marks]

a. Calculation: sum of incremental claims per column / sum of corresponding exposures.

	0	1	2	3	4
Claim ratio	48,5%	28,6%	4,8%	0,8%	0,8%

b. Calculation: sum of claim ratios for delays 0-4 in a.

Average	83,5 %

c. Calculation: Results from a) divided by result in b).

	0	1	2	3	4
Cumulative pi(cum.)	58,05 %	92,31 %	98,05 %	99,02 %	100,00 %
Incremental pi(inc.)	58,05 %	34,25 %	5,75 %	0,97 %	0,98 %

d. Calculation: Outstanding = Exposure x Loss ratio x (1-pi(cum.))

Accident year	Exposure	Developed to	Observed	pi(cum.)	Loss ratio	Outstanding
2007	122	4	91	100,0 %	83,5 %	0
2008	126	3	96	99,0 %	83,5 %	1
2009	148	2	123	98,1 %	83,5 %	2
2010	153	1	131	92,3 %	83,5 %	10
2011	167	0	85	58,1 %	83,5 %	58
Total	716		526			72

e. Calculation: Outstanding = Exposure x Loss ratio x pi(inc.)

	Development year				
Accident year	0	1	2	3	4
2 007					
2 008					1
2 009				1	1
2 010			7	1	1
2 011		48	8	1	1

2. Chain ladder method

a. Estimate the year-on-year development factors. [10 marks]

b. Estimate the payment pattern expressed in percent of ultimate cost. [10 marks]

c. Estimate the overall claim ratio for each accident year. [10 marks]

d. Estimate the outstanding claim payments for each accident year. [10 marks]

e. Fill the missing cells in the incremental run-off triangle with predictions. [10 marks]

a. Calculation: Sum of cumulative claims per column / sum of corresponding cumulative claims in previous column.

Empirical	0	1	2	3	4
Average		159,92 %	106,60 %	101,09 %	101,11 %

b. Calculation: Accumulate development factors to end, divide accumulated factors for each delay by ultimate

delta to pi	0	1	2	3	4
Devt. factor (incr.)		1,5992	1,0660	1,0109	1,0111
Devt. factor (cum.)	100 %	160 %	170 %	172 %	174 %
pi (cum.)	57,39 %	91,78 %	97,84 %	98,90 %	100,00 %
pi (incr.)	57,39 %	34,39 %	6,06 %	1,06 %	1,10 %

c. Calculation for c. Observed / (Exposure x pi(cum.))

d. Calculation for d. Calculation: Outstanding = Exposure x Loss ratio x (1-pi(cum.))

Accident year	Exposure	Developed to	Observed	pi(cum.)	c. Loss ratio	d. Outstanding
2007	122	4	91	100 %	74,6 %	0
2008	126	3	96	99 %	77,0 %	1
2009	148	2	123	98 %	84,9 %	3
2010	153	1	131	92 %	93,3 %	12
2011	167	0	85	57 %	88,7 %	63
Total	716		526		83,8 %	79

e. Calculation: Outstanding = Exposure x Loss ratio x pi(inc.)

	Development year				
Accident year	0	1	2	3	4
2007					
2008					1
2009				1	1
2010			9	2	2
2011		51	9	2	2

3. Benktander's method

With the payment pattern from question 2, apply Benktander's method to estimate the outstanding claim payments for each accident year. [10 marks]

Calculation: **Theta_CL = Observed / (Exposure x pi(cum.))**

Theta_BF = sum(observed) / sum(Exposure x pi(cum.))

Theta_bar = z x Theta_CL + (1-z) x Theta_BF

Outstanding = Exposure x Theta_bar x (1-pi(cum.))

Accident year	Exposure	Developed to	Observed	pi(cum.)	Theta_CL	Theta _BF	z	Theta_bar	Outstanding
2007	122	4	91	100 %	74,6 %	83,8 %	100 %	74,6 %	0
2008	126	3	96	99 %	77,0 %	83,8 %	99 %	77,1 %	1
2009	148	2	123	98 %	84,9 %	83,8 %	98 %	84,9 %	3
2010	153	1	131	92 %	93,3 %	83,8 %	92 %	92,5 %	12
2011	167	0	85	57 %	88,7 %	83,8 %	57 %	86,6 %	62
Total	716		526		83,8 %				77

Other questions

a. Would you characterize the payments as short-tailed or long-tailed? [10 marks]
b. You have used premium as a measure of risk exposure, but premium is not always a reliable measure. Suggest one other measure of risk exposure that you can easily use in this situation. [10 marks]

Answers:

a. Short-tailed

b. One could use payments in the accident year (at delay 0) to measure exposure

4. Generalised linear models

A generalised linear models has the structure $\mathrm{E}\left(X_{je} / p_j\right) = h\left(\mathbf{m}'_{je}\boldsymbol{\beta}\right)$, where h^{-1} is called the link function and $\mu_{je} = \mathbf{m}'_{je}\boldsymbol{\beta}$ is a linear function of unknown model parameters $\boldsymbol{\beta}$.

a. Explain how the assumptions underlying the Bornhuetter-Ferguson method can be expressed in a generalized linear model. [10 marks]
b. Explain how the assumptions underlying the Chain ladder method can be expressed in a generalized linear model. [10 marks]
c. Explain how you could implement a generalized linear model that allows you to estimate payment inflation along calendar years. [20 marks]

To answer a-c above, please specify the appropriate link function and the linear function of the model parameters. No calculations are required.

Answers:

Link function: logarithmic, giving multiplicative means.

a.

$$\mu_{je} = \alpha + \beta_e \,.$$

$$\begin{bmatrix} \mu_{10} \\ \mu_{11} \\ \mu_{12} \\ \cdots \\ \mu_{1,J-1} \\ \mu_{20} \\ \mu_{21} \\ \cdots \\ \mu_{2,J-2} \\ \cdots \end{bmatrix} = \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & \cdots & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & \cdots & 1 & 0 \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right] \begin{bmatrix} \alpha \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{J-2} \\ \beta_{J-1} \end{bmatrix}$$

b.

$$\mu_{je} = \alpha_j + \beta_e$$

$$\begin{bmatrix} \mu_{10} \\ \mu_{11} \\ \mu_{12} \\ \cdots \\ \mu_{1,J-1} \\ \mu_{20} \\ \mu_{21} \\ \cdots \\ \mu_{2,J-2} \\ \cdots \end{bmatrix} = \left[\begin{array}{cccccccccc} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ \hline 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right] \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_J \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{J-2} \\ \beta_{J-1} \end{bmatrix}$$

c.

$$\mu_{je} = \alpha + \beta_e + \lambda \cdot (j + e - 1) \,.$$

$$\begin{bmatrix} \mu_{10} \\ \mu_{11} \\ \mu_{12} \\ \cdots \\ \mu_{1,J-1} \\ \mu_{20} \\ \mu_{21} \\ \cdots \\ \mu_{2,J-2} \\ \cdots \end{bmatrix} = \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & \cdots & 0 & 0 & 2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & \cdots & 0 & 1 & J-1 \\ \hline 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & \cdots & 1 & 0 & J-1 \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right] \begin{bmatrix} \alpha \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{J-2} \\ \beta_{J-1} \\ \lambda \end{bmatrix}$$

5. Categories of outstanding claims

The abbreviations RBNS, IBNR and CBNI are common in loss reserving.

a. Give two possible reasons why a claim could be "RBNS" on a given date. [10 marks]

b. Give two possible reasons why a claim could be "IBNR" on a given date. [10 marks]

c. Why does an insurance company make a provision for claims CBNI? [10 marks]

a. Claims could be reported but not settled because of outstanding damage assessments, repairs, medical examinations, legal procedures, and so on. Any two sensible reasons will suffice, as long as the candidate understands that claims RBNS are reported claims.

b. Claims could be incurred but not reported because of the claimant being unaware of his/her right to claim, relying on other funding sources before insurance, administrative procedures, public holidays, and so on. The candidate should demonstrate an understanding that these are unreported claims.

c. An insurance company must make a provision for claims CBNI because claims against insurance policies issued before the balance date constitute a liability, much in the same way as claims IBNR. The candidate should ideally mention that a CBNI provision is not for all of next year's claims, only for those where policies already have been issued.